TEST 4

Math 104 **Score:** _____ out of 100

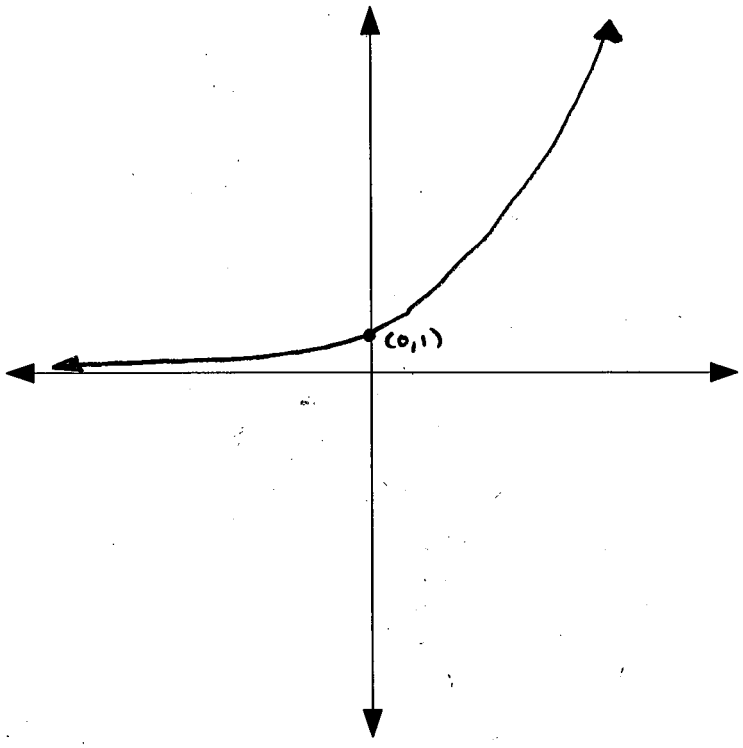
4/29/2013 **Name:** key

Read all of the following information before starting the exam:

- You have 50 minutes to complete the exam.
- Show all work (if necessary), clearly and in order, if you want to recieve full credit. Please make sure you read the directions for each problem. If the problem requires work I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Please box/circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 6 problems (and 1 bonus problem) and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- All answers must be exact, no rounding. Please keep things in fraction form whenever possible.
- Good luck!

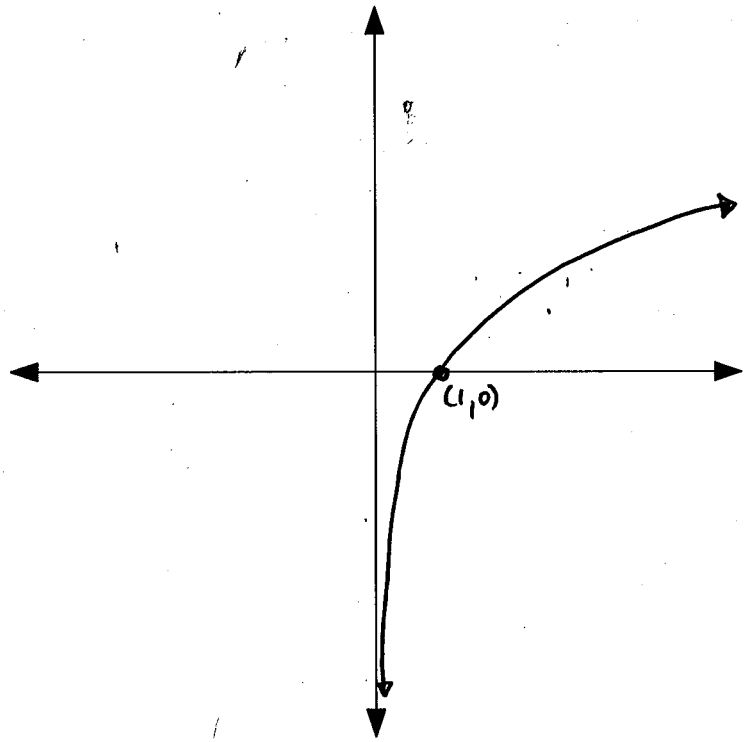
1. (a) Graph $y = 3^x$ by **filling in the table** and **plotting the points.**

x	y
-3	$3^{-3} = \frac{1}{27}$
-2	$3^{-2} = \frac{1}{9}$
-1	$3^{-1} = \frac{1}{3}$
0	$3^0 = 1$
1	$3^1 = 3$
2	$3^2 = 9$
3	$3^3 = 27$



(b) Graph $y = \log_3(x)$ by **filling in the table** and **plotting the points.**

x	y
$\frac{1}{27}$	-3
$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2
27	3



2. Solve for x.

(a) $6^x - 6 = 30$.

$6^x = 36$

$\boxed{x = 2}$

OR

$6^x = 36$

$\log_6(6^x) = \log_6(36)$

$\boxed{x = 2}$

(b) $3^{4x} = 81$.

$\log_3(3^{4x}) = \log_3(81)$

$4x = 4$

$\boxed{x = 1}$

(c) $7^{3x-4} = 49$.

$\log_7(7^{3x-4}) = \log_7(49)$

$3x - 4 = 2$

$3x = 6$

$\boxed{x = 2}$

OR

$7^{3x-4} = 7^2$

$3x - 4 = 2$

$3x = 6$

$\boxed{x = 2}$

(d) $3^{3x-4} = 3^{2x+9}$.

$3x - 4 = 2x + 9$

$3x - 2x = 13$

$\boxed{x = 13}$

OR

$\log_3(3^{3x-4}) = \log_3(3^{2x+9})$

$3x - 4 = 2x + 9$

$\boxed{x = 13}$

(e) $5(1.08)^x = 200$.

$1.08^x = \frac{200}{5} = 40$

$1.08^x = 40$

$\log_{1.08}(1.08^x) = \log_{1.08}(40)$

$\boxed{x = \log_{1.08}(40) = \frac{\ln(40)}{\ln(1.08)} \approx 47.9318}$

3. Determine the future value of a principal (**rounded to the nearest cent**) of \$5000 invested at 7.5% for 41 years compounded:

(a) Annually

$$A = P(1+r)^t$$

$$A = 5000(1+0.075)^{41}$$

$$A = 5000(1.075)^{41}$$

$$A \approx \boxed{\$96987.78}$$

(b) Monthly

$$A = P\left(1+\frac{r}{m}\right)^{tm}$$

$$A = 5000\left(1+\frac{0.075}{12}\right)^{(41)(12)}$$

$$A \approx \boxed{\$107,218.46}$$

(c) Continuously

$$A = Pe^{rt}$$

$$A = 5000\,e^{0.075(41)} \approx \boxed{\$108,249.41}$$

4. How much principal (**rounded to the nearest cent**) do you need to invest at 5.5% compounded continuously for 15 years to make a future value of \$9000?

$$A = Pe^{rt}$$
$$9000 = Pe^{0.055(15)}$$
$$P = \frac{9000}{e^{0.055(15)}} \approx \boxed{\$3944.11}$$

5. How long (**rounded to the nearest hundredth**) would you have to invest \$6,000 at 5.5% compounded continuously to make a future value of \$20,000?

$$A = Pe^{rt}$$
$$20000 = 6{,}000e^{0.055t}$$
$$\frac{20{,}000}{6{,}000} = e^{0.055t}$$
$$\ln\left(\frac{20{,}000}{6{,}000}\right) = \ln\left(e^{0.055t}\right)$$
$$\ln\left(\frac{20}{6}\right) = 0.055t$$
$$t = \frac{\ln(20/6)}{0.055}$$
$$\boxed{t \approx 21.89 \text{ years}}$$

6. Suppose at the beginning of each year you add \$3000 to an account invested at 7% compounded continuously. How much do you have (**rounded to the nearest cent**) after

(a) 2 years

$$3000e^{0.07(1)} + 3000e^{0.07(2)} \approx \boxed{\$6{,}668.35}$$

$3000e^{0.07(1)}$: this is \$3000 that is invested for only 1 year.

$3000e^{0.07(2)}$: this is \$3000 invested for 2 years.

(b) 3 years

$$3000e^{0.07(1)} + 3000e^{0.07(2)} + 3000e^{0.07(3)} \approx \boxed{\$10{,}369.38}$$

$3000e^{0.07(1)}$: this is \$3000 invested for 1 year.

$3000e^{0.07(2)}$: this is \$3000 invested for 2 years.

$3000e^{0.07(3)}$: this is \$3000 invested for 3 years.

7. ♠ Solve for x. Only an exact answer will be accepted and you must show all work:

$$2 \cdot 6^x = 7^x$$

SOL 1

$$2 = \frac{7^x}{6^x} = \left(\frac{7}{6}\right)^x$$

$$\log_{7/6}(2) = \log_{7/6}\left(\frac{7}{6}^x\right)$$

$$\boxed{\log_{7/6}(2) = x}$$

$$x = \frac{\ln(2)}{\ln(7/6)}$$

SAME.

SOL 2

$$\ln(2 \cdot 6^x) = \ln(7^x)$$

$$\ln(2) + \ln(6^x) = \ln(7^x)$$

$$\ln(2) + x\ln(6) = x\ln(7)$$

$$\ln(2) = x(\ln(7) - \ln(6))$$

$$\boxed{x = \frac{\ln(2)}{\ln(7) - \ln(6)}}$$

Many other methods as well...