TEST 1

Math 152 - Calculus II

Score: ________ out of 100

9/21/2012

Name: Key

Read all of the following information before starting the exam:

- You have 50 minutes to complete the exam.
- Show all work, clearly and in order, if you want to get full credit. Please make sure you read the directions for each problem. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Please box/circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 7 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. Find the average value of $f(x) = \sin(2x)$ on $[0, \pi/4]$.

$$\frac{1}{\frac{\pi}{4} - 0} \int_0^{\pi/4} \sin(2x)\,dx$$

$$u = 2x \Rightarrow \begin{array}{l} u(0) = 2 \cdot 0 = 0 \\ u(\pi/4) = 2 \cdot \frac{\pi}{4} = \frac{\pi}{2} \end{array}$$

$$\frac{du}{dx} = 2 \Rightarrow dx = \frac{du}{2}$$

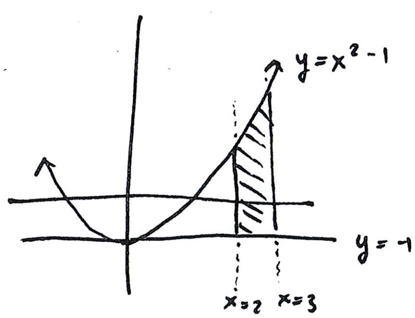
$$= \frac{4}{\pi} \int_0^{\pi/2} \sin(u) \cdot \frac{du}{2} = \frac{2}{\pi} \int_0^{\pi/2} \sin(u)\,du$$

$$= \frac{2}{\pi} \left[-\cos(u) \right]_0^{\pi/2}$$

$$= \frac{2}{\pi} \left[-\cos\left(\frac{\pi}{2}\right) - (-\cos(0)) \right]$$

$$= 2/\pi \left[-0 + 1 \right] = \boxed{2/\pi}$$

2. Find the area enclosed by the curves $y = x^2 - 1$, $y = -1$, $x = 2$ and $x = 3$.



$$\int_2^3 [(x^2 - 1) - (-1)]\,dx = \int_2^3 [x^2 - 1 + 1]\,dx$$

$$= \int_2^3 [x^2]\,dx$$

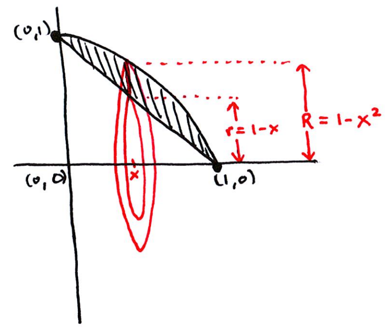
$$= \left[\frac{x^3}{3} \right]_2^3$$

$$= \frac{27}{3} - \frac{8}{3} = \boxed{\frac{19}{3}}$$

3. **Set up but do not evaluate the integral** for the volume of the solid obtained by rotating the region bounded by $y = 1 - x^2$ and $y = 1 - x$ about the x-axis.

Best Method : washer / disk

Picture :



points of intersection :

$1 - x^2 = 1 - x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0$ OR $x = 1$

variable of integration : x

inner radius: $r = 1 - x$

outer radius : $R = 1 - x^2$

$a = 0$

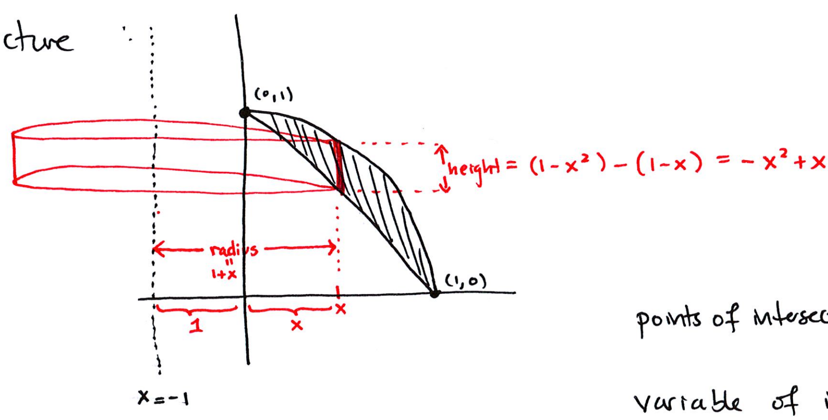
$b = 1$

$$\boxed{\text{Volume} = \int_0^1 \pi(1-x^2)^2 - \pi(1-x)^2 \, dx}$$

4. **Set up but do not evaluate the integral** for the volume of the solid obtained by rotating the region bounded by $y = 1 - x^2$ and $y = 1 - x$ about the line $x = -1$.

Best Method : cylindrical shells

Picture :



points of intersection : $x = 0$ OR $x = 1$ (see (3))

variable of integration : x

radius : $1 + x$

height : $-x^2 + x$

$a = 0$

$b = 1$

$$\boxed{\text{Volume} = \int_0^1 2\pi(1+x)(-x^2+x)\,dx}$$

5. Set up but do not evaluate the integral for the length of the curve $y = \sin(x)$ from $x = 0$ to $x = 3\pi$.

Arc length of $y = f(x)$ from $x=a$ to $x=b$ $= \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

$f(x) = \sin(x)$
$f'(x) = \cos(x)$

so

Arc length of $y = \sin(x)$ from $x=0$ to $x = 3\pi$ $= \boxed{\int_0^{3\pi} \sqrt{1 + [\cos(x)]^2}\, dx}$

6. A force of 10 N is required to hold a spring that has been stretch from its natural length of 0.2 m to a length of 0.3 m. How much work is done in stretching the spring from 0.3 m to 0.4 m?

$F = kx$

$F = 10N = k(0.3 - 0.2) = k(0.1) \Rightarrow k = 100$

$W = \int_a^b F(x)\, dx$

Here $a = 0.3 - \underbrace{0.2}_{\text{natural length of spring}} = 0.1$

$b = 0.4 - 0.2 = 0.2$

so

$W = \int_{0.1}^{0.2} 100x\, dx = \left[\frac{100x^2}{2}\right]_{0.1}^{0.2}$

$= 50\left((0.2)^2 - (0.1)^2\right)$

$= 50 \cdot 0.03$

$= \boxed{1.5\ J}$

PICK ONE OF THE FOLLOWING:

7. Please indicate which one you do NOT want me to grade by putting an X through it, otherwise I will grade the first one worked on:

(a) Evaluate $\int \tan^{-1}(x)dx$.

Integration By parts: LIATE ! so

$\underset{\tan^{-1}(x)}{\uparrow}$

$\int \tan^{-1}(x)\,dx$

$u = \tan^{-1}(x)$ || $dv = 1\,dx$

$du = \frac{1}{1+x^2}dx$ || $v = x$.

$= x\tan^{-1}(x) - \int x \cdot \frac{1}{1+x^2}dx$

$= x\tan^{-1}(x) - \int \frac{x}{1+x^2}dx$ ← now use a substitution (since we already used the variable u, I will use t)

$t = 1+x^2$

$\frac{dt}{dx} = 2x \Rightarrow dx = \frac{dt}{2x}$

$= x\tan^{-1}(x) - \int \frac{\cancel{x}}{t}\cdot\frac{dt}{2\cancel{x}}$

$= x\tan^{-1}(x) - \frac{1}{2}\int \frac{1}{t}dt = x\tan^{-1}(x) - \frac{1}{2}\ln|t| + C$

$= \boxed{x\tan^{-1}(x) - \frac{1}{2}\ln|1+x^2| + C}$ OR

$= \boxed{x\tan^{-1}(x) - \frac{1}{2}\ln(1+x^2) + C}$ since $1+x^2 \geq 0$

(b) Evaluate $\int x^2\cos(x)dx$.

LIATE !

x^2 $\cos(x)$

So

$u = x^2$ || $dv = \cos(x)\,dx$

$du = 2x\,dx$ || $v = \sin(x)$

$\int x^2\cos(x)\,dx = x^2\sin(x) - \int \sin(x)\cdot 2x\,dx$

$= x^2\sin(x) - 2\underbrace{\int \sin(x)\cdot x\,dx}_{\text{another integration by parts: LIATE}}$

x $\sin(x)$

$u = x$ || $dv = \sin(x)dx$

$du = 1\,dx$ || $v = -\cos(x)$

$= x^2\sin(x) - 2\left[x(-\cos(x)) - \int(-\cos(x))\,dx\right]$

$= x^2\sin(x) + 2x\cos(x) - 2\int\cos(x)\,dx$

$= \boxed{x^2\sin(x) + 2x\cos(x) - 2\sin(x) + C}$