Math 152 - Fall 2012 - Quiz #10 **Score:** ________ out of 10.

Name: Key

SOLVE ONE SIDE OF THE FOLLOWING:

Please indicate which side you do NOT want me to grade by putting an X through it, otherwise I will grade the first side worked on:

Show all work clearly and in order. Please box your answers.

1. Using the formula, set up a table and find the first FOUR nonzero terms of the Maclaurin series for

$$f(x) = \frac{1}{1+2x} = (1+2x)^{-1}.$$

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	$f(x) = (1+2x)^{-1}$	$(1+0)^{-1} = 1$
1	$f'(x) = -1(1+2x)^{-2}(2) = -2(1+2x)^{-2}$	$-2(1+0)^{-2} = -2$
2	$f''(x) = -1(-2)(1+2x)^{-3}(2)(2) = 8(1+2x)^{-3}$	$8(1+2\cdot 0)^{-3} = 8$
3	$f'''(x) = -1(-2)(-3)(1+2x)^{-4}(2)(2)(2) = -48(1+2x)^{-4}$	$-48(1+0)^{-4} = -48$

Maclaurin Series: $f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots =$

$$\boxed{1 + \frac{-2}{1!}x + \frac{8}{2!}x^2 + \frac{-48}{3!}x^3 + \cdots}$$

2. Using the formula, set up a table and find the first TWO nonzero terms of the Taylor series about $x_0 = 1$ for

$$f(x) = \sin\left(\frac{\pi}{2}x\right).$$

n	$f^{(n)}(x)$	$f^{(n)}(1)$
0	$f(x) = \sin\left(\frac{\pi}{2}x\right)$	$\sin\left(\frac{\pi}{2}\right) = 1$
1	$f'(x) = \cos\left(\frac{\pi}{2}x\right)\cdot\frac{\pi}{2}$	$\cos\left(\frac{\pi}{2}\right)\frac{\pi}{2} = 0\cdot\frac{\pi}{2} = 0$
2	$f''(x) = -\sin\left(\frac{\pi}{2}x\right)\cdot\frac{\pi}{2}\cdot\frac{\pi}{2}$	$-\sin\left(\frac{\pi}{2}\right)\cdot\frac{\pi}{2}\cdot\frac{\pi}{2} = -\frac{\pi^2}{4}$

Taylor series about $x_0 = 1$: $f(1) + \frac{f'(1)}{1!}(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \cdots =$

$$1 + \frac{0}{1!}(x-1) + \frac{\left(-\frac{\pi^2}{4}\right)}{2!}(x-1)^2 + \cdots =$$

$$\boxed{1 - \frac{\pi^2}{8}(x-1)^2 + \cdots}$$

3. Find the radius and interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(x-9)^n}{n}$$

using the Ratio Test for Absolute Convergence:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{(x-9)^{n+1}}{n+1}}{\frac{(x-9)^n}{n}}\right| = \lim_{n\to\infty}\left|\frac{(x-9)^{n+1}}{n+1}\cdot\frac{n}{(x-9)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{\cancel{(x-9)^n}(x-9)\cdot n}{(n+1)\cancel{(x-9)^n}}\right|$$

$$= \lim_{n\to\infty}\frac{|x-9|\,n}{n+1}$$

$$= |x-9|\lim_{n\to\infty}\frac{n}{n+1}$$

$$= |x-9|\lim_{n\to\infty}\frac{n}{(n+1)}\frac{(1/n)}{(1/n)}$$

$$= |x-9|\lim_{n\to\infty}\frac{1}{1+1/n}$$

$$= |x-9|\left(\frac{1}{1+0}\right)$$

$$= |x-9|\cdot 1$$

$$= |x-9|$$

so the power series converges when $|x-9| < 1$

That is, $-1 < x-9 < 1$
That is, $-1+9 < x < 1+9$
That is, $8 < x < 10$

the power series diverges when $|x-9| > 1$
the power series may converge or diverge when $|x-9| = 1$

Test the endpoints of the interval $8 < x < 10$:

If $x=8$ plug $x=8$ into the power series to get $\displaystyle\sum_{n=1}^{\infty}\frac{(8-9)^n}{n} = \sum_{n=1}^{\infty}\frac{(-1)^n}{n}$

This is an alternating series so let's try the Alternating Series Test:

Check (i) the sequence $\{\frac{1}{n}\}$ is decreasing :

<u>3 Methods!</u> I will embed it into a function

$f(x) = \frac{1}{x} \Rightarrow f'(x) = \frac{-1}{x^2} < 0$ negative positive

for $x \geq 1$

so $f(x)$ is decreasing (strictly)

Hence, $\{\frac{1}{n}\}$ is decreasing ✓

(ii) $\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

so <u>converges</u> when $x = 8$

<u>If $x = 10$</u> plug $x = 10$ into the power series to get $\sum_{n=1}^{\infty} \frac{(10-9)^n}{n} = \sum_{n=1}^{\infty} \frac{1^n}{n}$

$= \sum_{n=1}^{\infty} \frac{1}{n}$

harmonic series
(or p-series with $p = 1 \leq 1$)
<u>diverges</u> when $x = 10$.

interval of convergence : $\boxed{8 \leq x < 10}$ or $\boxed{[8, 10)}$

radius of convergence : $\boxed{R = 1}$