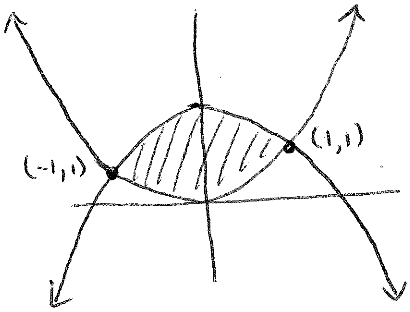
Math 152 - Fall 2012 - Quiz #2

Score: ________ out of 10.

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Find the area of the region bounded by $y = x^2$ and $y = 2 - x^2$.



points of intersection: $x^2 = 2 - x^2$

$2x^2 = 2$

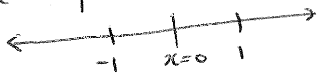
$x^2 = 1$

$x = \pm\sqrt{1} = \pm 1$

which function is above the other?

Method 1. (see picture) $2-x^2$ is above x^2 on $[-1,1]$

Method 2.



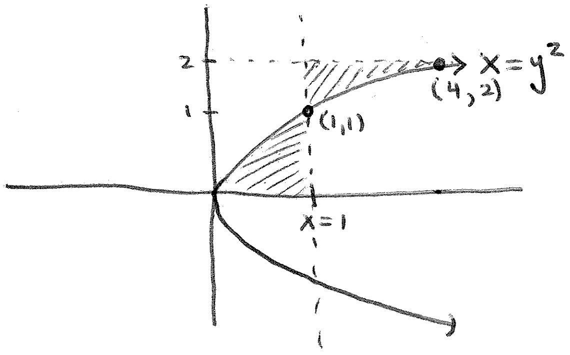
at $x=0$: $(0)^2 = 0$

$2-(0)^2 = 2$ so $2-x$ is above x^2 on $[-1,1]$

Integrate: $\int_{-1}^{1} [(2-x^2)-x^2]\,dx = \int_{-1}^{1} [2-2x^2]\,dx$

$= 2\int_{-1}^{1} (1-x^2)\,dx = 2\left[x - \frac{x^3}{3}\right]_{-1}^{1} = 2\left(\left(1-\frac{1^3}{3}\right) - \left((-1) - \frac{(-1)^3}{3}\right)\right) = \boxed{\frac{8}{3}}$

2. Find the area of the region bounded by $x = y^2$, $x = 1$ and $0 \le y \le 2$.



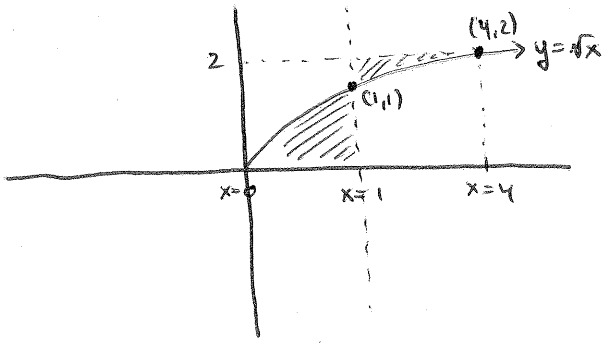
SOLUTION 1: (integration with respect to y):

$$A = \int_0^1 (1-y^2)\,dy + \int_1^2 (y^2-1)\,dy$$

$$= \left[y - \frac{y^3}{3}\right]_0^1 + \left[\frac{y^3}{3} - y\right]_1^2$$

$$= \left(\left(1-\frac{1}{3}\right)-0\right) + \left(\left(\frac{2^3}{3}-2\right) - \left(\frac{1}{3}-1\right)\right)$$

$$= \boxed{2}$$



SOLUTION 2: (Integration with respect to x)

$$A = \int_0^1 \sqrt{x}\,dx + \int_1^4 (2-\sqrt{x})\,dx$$

$$= \left[\frac{x^{3/2}}{3/2}\right]_0^1 + \left[2x - \frac{x^{3/2}}{3/2}\right]_1^4$$

$$= \left(\left(\frac{1}{3/2} - 0\right)\right) + \left(\left(2\cdot 4 - \frac{4^{3/2}}{3/2}\right) - \left(2 - \frac{1}{3/2}\right)\right)$$

$$= \boxed{2}$$