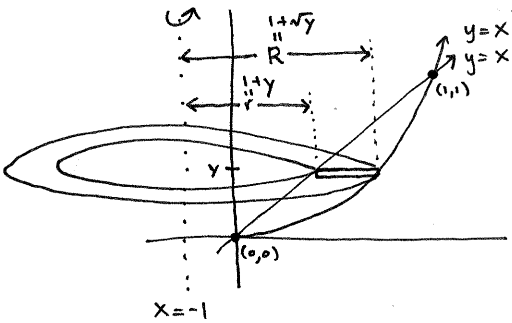
Math 152 - Fall 2012 - Quiz #3 Score: ________ out of 10.

Name: ______________________________

Show all work clearly and in order. Please box your answers. 10 minutes.

1. **Set up but do not evaluate the integral** for the volume of the solid obtained by rotating the region bounded by $y = x$ and $y = x^2$ about the line $x = -1$.

Washer/Disk Method:



$y=x^2 \Rightarrow x = \pm\sqrt{y} \Rightarrow x=\sqrt{y}$

$y=x \Rightarrow x=y$

variable of integration : y

points of intersection : $x = x^2 \Rightarrow x^2 - x = 0 \Rightarrow x(x-1)=0$

$x=0$ OR $x=1$

so $y=0$ OR $y=1$

so $(0,0)$ and $(1,1)$ (c, d)

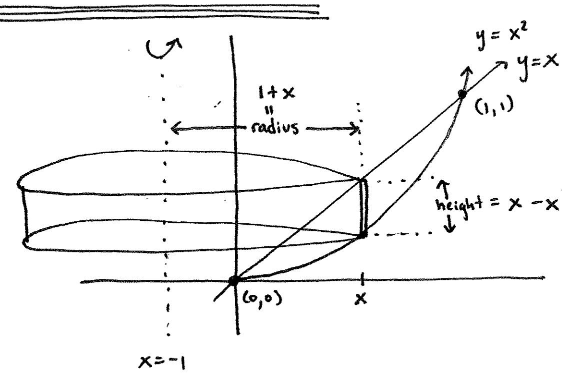
limits of integration: $c=0$, $d=1$

inner radius : $r = 1+y$

outer radius : $R = 1+\sqrt{y}$

$$V = \int_c^d (\pi R^2 - \pi r^2)\,dy = \boxed{\int_0^1 [\pi(1+\sqrt{y})^2 - \pi(1+y)^2]\,dy}$$

(Cylindrical) Shell Method



variable of integration : x

points of intersection : $(0,0)$ and $(1,1)$ (a, b)

limits of integration : $a=0$, $b=1$

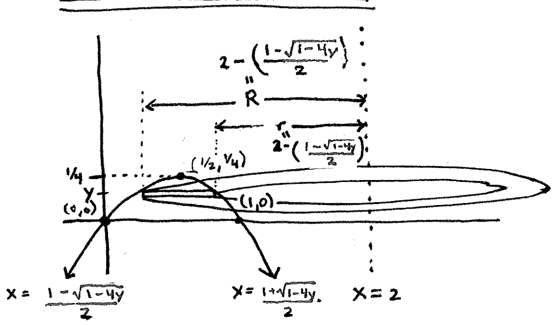
radius $= r = 1+x$

height $= h = x - x^2$

$$V = \int_a^b 2\pi \cdot r \cdot h\,dx = \boxed{\int_0^1 2\pi(1+x)(x-x^2)\,dx}$$

2. **Set up but do not evaluate the integral** for the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$.

Washer / Disk Method: (VERY HARD)



$y = x - x^2 \Rightarrow x^2 - x + y = 0 \Rightarrow x = \dfrac{+1 \pm \sqrt{1-4(1)(+y)}}{2} = \dfrac{1 \pm \sqrt{1-4y}}{2}$

so $x = \dfrac{1+\sqrt{1-4y}}{2}$ and $x = \dfrac{1-\sqrt{1-4y}}{2}$

Variable of integration: y

points of intersection: for $y = x - x^2$ and $y=0 \Rightarrow x(1-x)=0$

$x=0 \mid x=1$

so $(0,0)$ and $(1,0)$

limits of integration: $y=0=c$ and $y=\frac{1}{4}=d$ ← this is the largest y coordinate on the region

you can find the largest y coordinate two ways: Calc I: $y' = 1-2x$

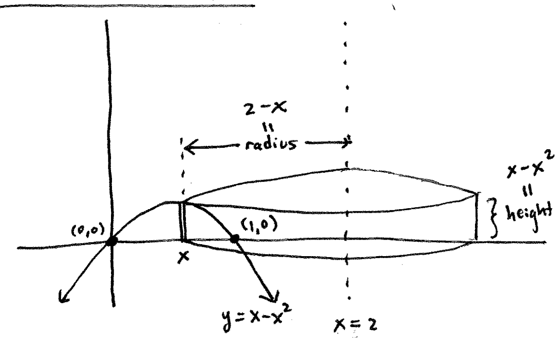
$y' = 0 = 1-2x$, $x = 1/2$ (critical pt.)

y': $+$ $-$ at $1/2$, so MAX, so $y = (1/2)-(1/2)^2$, $y = 1/4$

OR axis of symmetry :

for parabola $y = -x^2+x$

(Cylindrical) Shell Method



variable of integration : x

points of intersection : $(0,0)$ and $(1,0)$

limits of integration : $a=0$ and $b=1$

radius $= r = 2-x$

height $= h = x - x^2$

$$V = \int_a^b 2\pi \cdot r \cdot h\,dx = \boxed{\int_0^1 2\pi(2-x)(x-x^2)\,dx}$$

$$x = \frac{-b}{2a}$$

$$x = \frac{-1}{2(-1)} = \frac{1}{2}$$

so $y = 1/4$.

inner radius : $r = 2 - \left(\frac{1 + \sqrt{1-4y}}{2}\right)$

outer radius : $R = 2 - \left(\frac{1 - \sqrt{1-4y}}{2}\right)$

$$V = \int_c^d (\pi R^2 - \pi r^2)\, dy$$

$$= \boxed{\int_0^{1/4} \left[\pi\left(2 - \left(\frac{1-\sqrt{1-4y}}{2}\right)\right)^2 - \pi\left(2 - \left(\frac{1+\sqrt{1-4y}}{2}\right)\right)^2\right] dy}$$