Math 152 - Fall 2012 - Quiz #4 Score: ________ out of 10.

Name: key

Show all work clearly and in order. Please box your answers. 10 minutes.

SOLVE 2 OF THE FOLLOWING INTEGRALS:

Please indicate which one you do NOT want me to grade by putting an X through it, otherwise I will grade the two worked on:

1. Evaluate $\int \sin^3(4x)dx$.

$$\int \sin^3(4x)dx = \int \sin(4x)\sin^2(4x)\,dx$$

$\cos^2\theta + \sin^2\theta = 1$

$$= \int \sin(4x)\left(1-\cos^2(4x)\right)dx$$

Let $u = \cos(4x) \Rightarrow \frac{du}{dx} = -4\sin(4x)$

$\Rightarrow dx = \frac{du}{-4\sin(4x)}$

$$= \int \cancel{\sin(4x)}(1-u^2)\frac{du}{-4\cancel{\sin(4x)}}$$

$$= \frac{-1}{4}\int(1-u^2)\,du = -\frac{1}{4}\left[u - \frac{u^3}{3}\right] + C$$

$$= -\frac{1}{4}\left[\cos(4x) - \frac{\cos^3(4x)}{3}\right] + C$$

$$= \boxed{\frac{-\cos(4x)}{4} + \frac{\cos^3(4x)}{12} + C}$$

2. Evaluate $\int \cos^4(x)dx$.

$\cos^2\theta = \frac{1}{2}(1+\cos(2\theta))$

$$\int \cos^4(x)dx = \int[\cos^2(x)]^2dx = \int\left[\tfrac{1}{2}(1+\cos(2x))\right]^2dx = \int\left[\tfrac{1}{2}(1+\cos(2x))\right]\left[\tfrac{1}{2}(1+\cos(2x))\right]dx =$$

$\cos^2\theta = \frac{1}{2}(1+\cos(2\theta))$ with $\theta = 2x$

$$= \frac{1}{4}\int\left(1 + 2\cos(2x) + \cos^2(2x)\right)dx = \frac{1}{4}\int\left(1 + 2\cos(2x) + \tfrac{1}{2}[1+\cos(4x)]\right)dx =$$

$$= \frac{1}{4}\int\left(1 + 2\cos(2x) + \tfrac{1}{2} + \tfrac{1}{2}\cos(4x)\right)dx = \frac{1}{4}\int\left(\tfrac{3}{2} + 2\cos(2x) + \tfrac{1}{2}\cos(4x)\right)dx =$$

$$= \frac{1}{4}\left[\frac{3}{2}x + \frac{2\sin(2x)}{2} + \frac{1}{2}\cdot\frac{\sin(4x)}{4}\right] + C = \boxed{\frac{3}{8}x + \frac{\sin(2x)}{4} + \frac{\sin(4x)}{32} + C}$$

3. Evaluate $\int \tan^5(x)\sec^4(x)dx$.

SOLUTION 1:

$\sec^2\theta = 1 + \tan^2\theta$

$$\int \tan^5(x)\sec^4(x)dx = \int \tan^5(x)\sec^2(x)\cdot\sec^2(x)dx = \int \tan^5(x)\left[\tan^2(x)+1\right]\sec^2(x)\,dx$$

Let $u = \tan(x) \Rightarrow \frac{du}{dx} = \sec^2(x) \Rightarrow dx = \frac{du}{\sec^2(x)}$

$$= \int u^5[u^2+1]\cancel{\sec^2(x)}\cdot\frac{du}{\cancel{\sec^2(x)}} = \int(u^7+u^5)\,du = \frac{u^8}{8} + \frac{u^6}{6} + C$$

$$= \boxed{\frac{\tan^8(x)}{8} + \frac{\tan^6(x)}{6} + C}$$

SOLUTION 2

SOLUTION 2 :

$$\int \tan^5(x)\sec^4(x)\,dx = \int \tan^4(x)\sec^3(x)\cdot\sec(x)\tan(x)\,dx$$

$$= \int [\sec^2(x)-1]^2\sec^3(x)\cdot\sec(x)\tan(x)\,dx$$

$$u=\sec(x) \Rightarrow \frac{du}{dx}=\sec(x)\tan(x)$$

$$\Rightarrow dx = \frac{du}{\sec(x)\tan(x)}$$

$$= \int (u^2-1)^2u^3\cdot\cancel{\sec(x)\tan(x)}\cdot\frac{du}{\cancel{\sec(x)\tan(x)}}$$

$$= \int (u^2-1)(u^2-1)u^3\,du$$

$$= \int [u^4-2u^2+1]u^3\,du$$

$$= \int (u^7-2u^5+u^3)\,du$$

$$= \frac{u^8}{8}-\frac{2u^6}{6}+\frac{u^4}{4}+C$$

$$= \boxed{\frac{\sec^8(x)}{8}-\frac{\sec^6(x)}{3}+\frac{\sec^4(x)}{4}+C}$$