Math 152 - Fall 2012 - Quiz #6 **Score:** _____ out of 10.

Name: Key

Show all work clearly and in order. Please box your answers.

1. Write out the FORM of the partial fraction decomposition for the following (DO NOT find the numerical values for the unknown coefficients).

(a) $\dfrac{-3x^3+x^2-2x+6}{x^4+x^2} = \dfrac{-3x^3+x^2-2x+6}{x^2(x^2+1)} = \boxed{\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{Cx+D}{x^2+1}}$

(b) $\dfrac{4x^2-12x+1}{(x^2+4)(x-1)x^2} = \boxed{\dfrac{Ax+B}{x^2+4}+\dfrac{C}{x-1}+\dfrac{D}{x}+\dfrac{E}{x^2}}$

2. Evaluate $\displaystyle\int \frac{x+1}{x^2-4x+3}dx$.

Step 1: the degree of $x+1$ is 1; the degree of x^2-4x+3 is 2 } so we do NOT need to do long division! ✓

Step 2: $x^2-4x+3 = (x-3)(x-1)$

Step 3:

$$\frac{x+1}{x^2-4x+3} = \frac{x+1}{(x-3)(x-1)} = \frac{A}{x-1}+\frac{B}{x-3}$$

↑ ↑ Distinct Linear Factors! (CASE 1)

now find A and B:

$$\frac{x+1}{(x-3)(x-1)} = \frac{A(x-3)}{(x-1)(x-3)} + \frac{B(x-1)}{(x-3)(x-1)}$$

$$x+1 = A(x-3)+B(x-1)$$

$$x+1 = Ax-3A+Bx-B$$

$$1\cdot x+1 = (A+B)x \quad -3A-B$$

$A+B=1$

$A=1-B$ →

$-3A-B=1$

$-3(1-B)-B=1$

$-3+3B-B=1$

$-3+2B=1$

$2B=4$

$B=2$

$A=1-2=-1$

Hence,

$$\frac{x+1}{(x-3)(x-1)} = \frac{-1}{x-1}+\frac{2}{x-3}$$

Now,

$$\int \frac{x+1}{x^2-4x+3} = \int \left(\frac{-1}{x-1} + \frac{2}{x-3} \right) dx$$

$$= \int \frac{-1}{x-1} dx + \int \frac{2}{x-3} dx$$

$u = x-1$ $\quad$ $t = x-3$

$\frac{du}{dx} = 1 \Rightarrow dx = du$ $\quad$ $\frac{dt}{dx} = 1 \Rightarrow dx = dt$

$$= \int \frac{-1}{u} \cdot du + \int \frac{2}{t} \cdot dt$$

$$= -\ln|u| + 2\ln|t| + C$$

$$= \boxed{-\ln|x-1| + 2\ln|x-3| + C}$$