Math 152 - Fall 2012 - Quiz #7

Score: ________ out of 10.

Name: Key

Show all work clearly and in order. Please box your answers.

1. Determine whether the sequence converges, and if so find its limit.

(a) $\left\{ \dfrac{2n^2+3}{3n^2-n} \right\}_{n=1}^{\infty}$

SOL 1

$$\lim_{n\to\infty} \frac{2n^2+3}{3n^2-n} = \lim_{n\to\infty} \frac{(2n^2+3)\left(\frac{1}{n^2}\right)}{(3n^2-n)\left(\frac{1}{n^2}\right)}$$

$$= \lim_{n\to\infty} \frac{2+\frac{3}{n^2}}{3-\frac{1}{n}}$$

$$= \frac{2+0}{3+0} = \boxed{\frac{2}{3}}$$

convergent

SOL 2 Limit of numerator and denominator both go to ∞.
Embed the sequence into $f(x) = \dfrac{2x^2+3}{3x^2-x}$

we are using L'Hôpital's Rule.

so

$$\lim_{x\to\infty} \frac{2x^2+3}{3x^2-x} \overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{d}{dx}(2x^2+3)}{\frac{d}{dx}(3x^2-x)}$$

$$= \lim_{x\to\infty} \frac{4x+0}{6x-1}$$

$$= \lim_{x\to\infty} \frac{4}{6} = \frac{4}{6} = \frac{2}{3}$$

Therefore, $\displaystyle\lim_{n\to\infty} \frac{2n^2+3}{3n^2-n} = \boxed{\frac{2}{3}}$

convergent

(b) $\left\{ \dfrac{n}{\ln(n)} \right\}_{n=2}^{\infty}$

SOL The limit of the numerator and denominator both go to ∞.
Embed the sequence into $f(x) = \dfrac{x}{\ln(x)}$

so

$$\lim_{x\to\infty} \frac{x}{\ln(x)} = \lim_{x\to\infty} \frac{\frac{d}{dx}(x)}{\frac{d}{dx}(\ln(x))}$$

$$= \lim_{x\to\infty} \frac{1}{\left(\frac{1}{x}\right)} = \lim_{x\to\infty} x = \infty$$

Hence, $\displaystyle\lim_{n\to\infty} \frac{n}{\ln(n)} = \boxed{\infty}$ diverges

NOTE: some of you argued $n > \ln(n)$ for $n \geq 1$ and so n grows faster than $\ln(n)$. You need to state why and show some work!

2. Show that the given sequence is strictly increasing or strictly decreasing.

$$\left\{ \frac{2n}{3n-1} \right\}_{n=1}^{\infty}$$

SOL 1

$a_{n+1} - a_n$

$$= \frac{2(n+1)}{3(n+1)-1} - \frac{2n}{3n-1}$$

$$= \frac{2n+2}{3n+2} - \frac{2n}{3n-1}$$

$$= \frac{(2n+2)(3n-1)-(2n)(3n+2)}{(3n+2)(3n-1)}$$

$$= \frac{-2}{\underbrace{(3n+2)}_{+\text{ for } n\geq 1}\underbrace{(3n-1)}_{+\text{ for } n\geq 1}} \leftarrow \text{always} -$$

$< 0 \leftarrow$ since $\frac{(-)}{(+)(+)} = (-)$

SOL 2

$$\frac{a_{n+1}}{a_n} = \frac{\frac{2(n+1)}{3(n+1)-1}}{\frac{2n}{3n-1}}$$

$$= \frac{2(n+1)}{(3n+2)} \frac{(3n-1)}{(2n)}$$

$$= \frac{3n^2+2n-1}{3n^2+2n} \leftarrow \text{numerator always 1 less than denominator}$$

< 1

SOL 3 Embed the sequence into

$$f(x) = \frac{2x}{3x-1}$$

Now $f'(x) = \dfrac{(3x-1)(2)-(2x)(3)}{(3x-1)^2}$

$$= \frac{6x-2-6x}{(3x-1)^2}$$

$$= \frac{-2}{(3x-1)^2} \begin{array}{l}\leftarrow \text{always} - \\ \leftarrow \text{always} + \text{ if } x \neq 1/3\end{array}$$

$< 0 \leftarrow$ since $\frac{(-)}{(+)} = (-)$

So $f(x)$ is strictly decreasing for $x \geq 1$.
Hence, the sequence is strictly decreasing for $n \geq 1$

The sequence is strictly $\boxed{\text{decreasing}}$