{Change to $\sum_{k=1}^{\infty} ar^{k-1}$}

Geometric Series: $\underbrace{\sum_{k=0} ar^k}$ = $\underbrace{\sum_{k=1} ar^{k-1}}$ = $a + ar + ar^2 + \dots$

try to change the sum into either FORM

{Change to $\sum_{k=0}^{\infty} ar^k$}

Math 152 - Fall 2012 - Quiz #8 **Score:** ________ out of 10.

Name: key

Show all work clearly and in order. Please box your answers.

1. Each series below is geometric. Determine both a and r. Then decide whether the series converges or diverges. If the series converges, then find its sum. If it diverges, write "NO SUM."

(a) $\sum_{k=1}^{\infty}\left(\frac{3}{4}\right)^{k-1} = \sum_{k=1}^{\infty} \underbrace{1}_{a} \cdot \underbrace{\left(\frac{3}{4}\right)}_{r}^{k-1}$

$a = \boxed{1}$

$r = \boxed{3/4}$

sum= $\frac{1}{1-3/4} = \frac{1}{1/4} = \boxed{4}$

(b) $\sum_{k=0}^{\infty}(-1)^{k+1}\frac{3^{2k}}{8^{k+1}} = \sum_{k=0}^{\infty} \frac{(-1)(-1)^k (3^2)^k}{8 \cdot 8^k}$

$= \sum_{k=0}^{\infty} \frac{(-1)(-1)^k}{8} \frac{(9)^k}{8^k}$

$= \sum_{k=0}^{\infty} \underbrace{\left(\frac{-1}{8}\right)}_{a} \underbrace{\left(\frac{-9}{8}\right)}_{r}^k$

$a = \boxed{-1/8}$

$r = \boxed{-9/8}$

sum= $\boxed{\text{NO SUM}}$

since $|r| = |-9/8| = 9/8 > 1$ divergent

2. Use the Divergence Test to determine whether the given series diverges. If the test yields no conclusion, then be sure to say so. You must set up, evaluate, and interpret the correct limit to earn credit.

$$\sum_{k=1}^{\infty}\frac{2k^2+3}{3k^3-2}$$

Here $a_k = \frac{2k^2+3}{3k^3-2}$

So $\lim_{k\to\infty} \frac{2k^2+3}{3k^3-2} = \lim_{k\to\infty} \frac{(2k^2+3)(\frac{1}{k^3})}{(3k^3-2)(\frac{1}{k^3})}$

$= \lim_{k\to\infty} \frac{\frac{2k^2}{k^3} + \frac{3}{k^3}}{\frac{3k^3}{k^3} - \frac{2}{k^3}} = \lim_{k\to\infty} \frac{2/k + 3/k^3}{3 - 2/k^3}$

$= \frac{0+0}{3-0} = 0$ $\boxed{\text{NO INFO}}$

3. Use the Integral Test to determine whether the given series converges or diverges. Clearly identify the function $f(x)$ you are embedding the sequence of terms into. You may assume that $f(x)$ is positive, decreasing and continuous for $x \geq 1$, so you do not need to verify this. Just use the integral test and state your conclusion.

$$\sum_{k=1}^{\infty}\frac{1}{k^2+4}$$

$f(x) = \frac{1}{x^2+4}$

$\int_1^{\infty} f(x)dx = \int_1^{\infty} \frac{1}{x^2+4}dx = \lim_{t\to\infty}\int_1^t \frac{1}{x^2+4}dx = \lim_{t\to\infty}\int_1^t \frac{1}{4(\frac{x^2}{4}+1)}dx$

$= \lim_{t\to\infty} \frac{1}{4}\int_1^t \frac{1}{(\frac{x}{2})^2+1}dx$

$u = \frac{x}{2} \Rightarrow u(1) = 1/2$, $u(t) = t/2$

$\frac{du}{dx} = \frac{1}{2} \Rightarrow dx = 2du$

$= \lim_{t\to\infty} \frac{1}{4}\int_{1/2}^{t/2} \frac{1}{u^2+1} \cdot 2du = \lim_{t\to\infty} \frac{1}{2}\left[\tan^{-1}(u)\right]_{1/2}^{t/2}$

$= \lim_{t\to\infty} \frac{1}{2}\left[\tan^{-1}(t/2) - \tan^{-1}(1/2)\right]$

$= \frac{1}{2}(\pi/2 - \tan^{-1}(1/2)) \approx 0.5536$

so $\boxed{\text{converges}}$