Math 152 - Fall 2012 - Quiz #9 **Score:** ________ out of 10.

Name: Key

Show all work clearly and in order. Please box your answers.

1. Use the ratio test to determine whether the series converges or diverges. If the test is inconclusive then say so.

$$\sum_{k=1}^{\infty} \underbrace{\frac{4^k}{k^2}}_{a_k}$$

$$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{4^{k+1}}{(k+1)^2}\cdot\frac{k^2}{4^k} = \lim_{k\to\infty} \frac{\cancel{4^k}\cdot 4\cdot k^2}{(k^2+2k+1)\cancel{4^k}}$$

$$= \lim_{k\to\infty} \frac{(4k^2)(\frac{1}{k^2})}{(k^2+2k+1)(\frac{1}{k^2})}$$

$$= \lim_{k\to\infty} \frac{4}{1+\frac{2}{k}+\frac{1}{k^2}} = \frac{4}{1+0+0} = 4 > 1$$

$\boxed{\text{diverges}}$

2. Use the root test to determine whether the series converges or diverges. If the test is inconclusive then say so.

$$\sum_{k=1}^{\infty} \underbrace{\left(\frac{2k^3-5}{5k^3-k}\right)^k}_{a_k}$$

$$\lim_{k\to\infty} (a_k)^{1/k} = \lim_{k\to\infty} \left(\left(\frac{2k^3-5}{5k^3-k}\right)^k\right)^{1/k} = \lim_{k\to\infty} \frac{(2k^3-5)(\frac{1}{k^3})}{(5k^3-k)(\frac{1}{k^3})}$$

$$= \lim_{k\to\infty} \frac{2-5/k^3}{5-1/k^2} = \frac{2}{5} < 1$$

$\boxed{\text{Converges}}$

3. Use the alternating series test to determine whether the series converges. If the test is inconclusive then say so.

$$\sum_{k=3}^{\infty} (-1)^k \frac{\ln(k)}{k}$$

① Show: $\left\{\frac{\ln(k)}{k}\right\}_{k=3}^{\infty}$ is a decreasing sequence.

3 Methods! Let's consider $f(x) = \frac{\ln(x)}{x}$ where $x \geq 3$.

$$f'(x) = \frac{x(\frac{1}{x}) - \ln(x)}{x^2} = \frac{1-\ln(x)}{x^2} < 0 \checkmark$$

$1-\ln(x)$ is negative since $x \geq 3 > e$

x^2 is positive since $x \geq 3$

so decreasing.

② Show: $\lim_{k\to\infty} \frac{\ln(k)}{k} = 0$. Consider $\lim_{x\to\infty} \frac{\ln(x)}{x} \overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0 \checkmark$ so $\boxed{\text{converges}}$