TEST 1

Math 152 - Calculus II | **Score:** ________ out of 100

9/20/2013 | **Name:** key

Read all of the following information before starting the exam:

- You have 50 minutes to complete the exam.
- Show all work, clearly and in order, if you want to get full credit. Please make sure you read the directions for each problem. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Please box/circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 10 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. Evaluate $\int \frac{e^x}{1-3e^x}dx$.

$u = 1-3e^x$

$\frac{du}{dx} = -3e^x \Rightarrow dx = \frac{du}{-3e^x}$

$$\int \frac{e^x}{u} \cdot \frac{du}{-3e^x} = -\frac{1}{3}\int \frac{1}{u}\,du = -\frac{1}{3}\ln|u| + C = \boxed{-\frac{1}{3}\ln|1-3e^x| + C}$$

2. Evaluate $\int \sec^2(2x+1)dx$.

$u = 2x+1$

$\frac{du}{dx} = 2 \Rightarrow dx = \frac{du}{2}$

$$\int \sec^2(u)\cdot\frac{du}{2} = \frac{1}{2}\int \sec^2(u)\,du = \frac{1}{2}\tan(u) + C$$

$$= \boxed{\frac{1}{2}\tan(2x+1) + C}$$

3. Suppose the average value of $f(x) = \sqrt{x}$ on $[0, b]$ is equal to 2. Find the value of b.

$$f_{ave} = 2 = \frac{1}{b-0}\int_0^b \sqrt{x}\,dx$$

$$2 = \frac{1}{b}\int_0^b x^{1/2}\,dx$$

$$2 = \frac{1}{b}\left[\frac{x^{3/2}}{(3/2)}\right]_0^b$$

$$2 = \frac{1}{b}\left(\frac{b^{3/2}}{(3/2)} - 0\right) = \frac{2}{3b}b^{3/2}$$

$$2 = \frac{2}{3}b^{3/2-1} = \frac{2}{3}b^{1/2}$$

$$\frac{2\cdot 3}{2} = b^{1/2}$$

$$3 = \sqrt{b}$$

$$\boxed{9 = b}$$

4. It takes 10 J of work to stretch a spring from its natural length to 0.5 m beyond its natural length. How much work is required to stretch the spring from 1 m beyond its natural length to 2 m beyond its natural length?

$$10 = \int_0^{0.5} kx\,dx = \left[\frac{kx^2}{2}\right]_0^{0.5} = \frac{k}{2}(0.5)^2 = \frac{k}{8}$$

$k = 80$

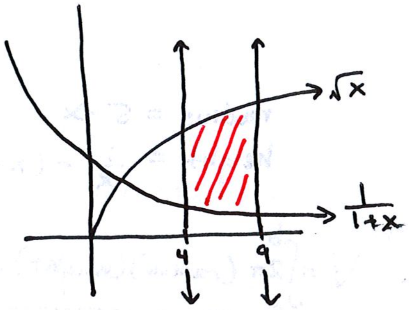
$$W = \int_1^2 kx\,dx = \int_1^2 80x\,dx = \left[\frac{80x^2}{2}\right]_1^2 = \frac{80}{2}(2^2-1^2)$$

$$= 40(4-1)$$

$$= 40(3)$$

$$= \boxed{120\text{ J}}$$

5. Find the area enclosed by the curves $y = \sqrt{x}$, $y = \dfrac{1}{1+x}$, $x = 4$ and $x = 9$.



$$A = \int_4^9 \left(\sqrt{x} - \frac{1}{1+x}\right) dx$$

$$= \int_4^9 \left(x^{1/2} - \frac{1}{1+x}\right) dx$$

$$= \int_4^9 x^{1/2}\,dx - \int_4^9 \frac{1}{1+x}\,dx$$

$$= \left[\frac{x^{3/2}}{(3/2)}\right]_4^9 - \int_4^9 \frac{1}{1+x}\,dx$$

$u = 1+x \Rightarrow \frac{du}{dx} = 1 \Rightarrow dx = du$

$u(4) = 1+4 = 5$

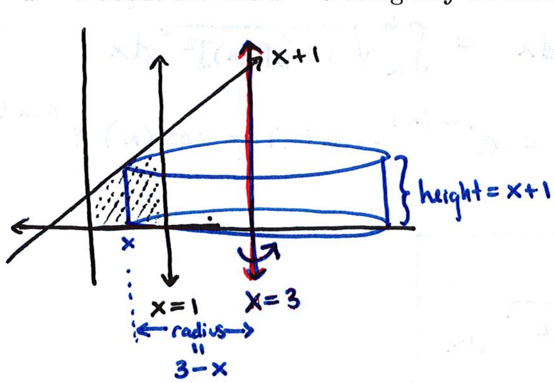
$u(9) = 1+9 = 10$

$$= \left[\frac{2}{3}\left(9^{3/2} - 4^{3/2}\right)\right] - \int_5^{10} \frac{1}{u}\,du = \frac{2}{3}(27-8) - \left[\ln|u|\right]_5^{10} = \frac{38}{3} - (\ln(10) - \ln(5)) = \boxed{\frac{38}{3} - \ln(2)}$$

$$\approx \boxed{11.974}$$

6. Find the volume of the solid obtained by rotating the region bounded by $y = x+1$, $y = 0$, $x = 0$ and $x = 1$ about the line $x = 3$ using **any method**.

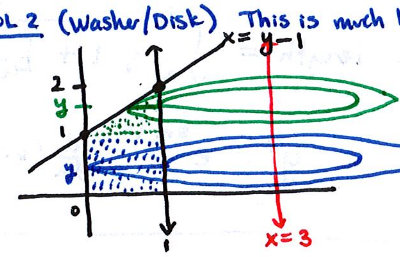
SOL 1 (shells)



$$V = \int_0^1 2\pi(3-x)(x+1)\,dx = \int_0^1 2\pi(3x - x^2 + x + 3)\,dx$$

$$= \int_0^1 2\pi(-x^2 + 2x + 3)\,dx = 2\pi\left[-\frac{x^3}{3} + \frac{2x^2}{2} + 3x\right]_0^1 = 2\pi\left(\frac{11}{3}\right) = \boxed{\frac{22\pi}{3}}$$

SOL 2 (Washer/Disk) This is much harder....



$r = 3-1 = 2$

$R = 3-(y-1) = 4-y$

$r = 3-1 = 2$

$R = 3$

pt. of intersection:

$y-1 = 1$

$y = 2$

$$V = V_1 + V_2$$

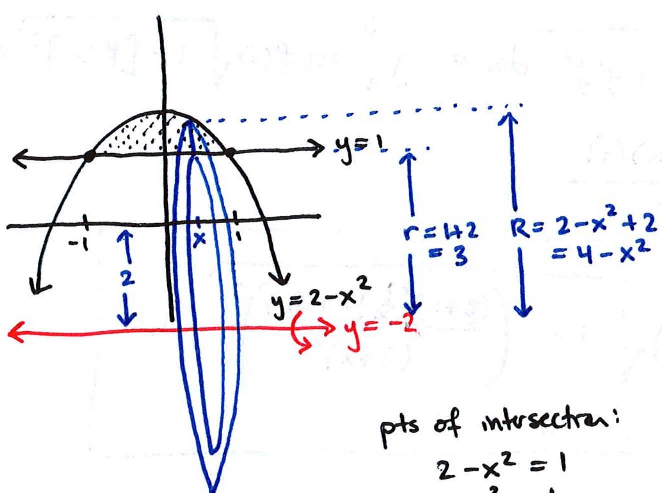
$$= \int_1^2 \pi(4-y)^2 - \pi(2)^2\,dy + \int_0^1 \pi(3)^2 - \pi(2)^2\,dy$$

$$= \pi\int_1^2 (16 - 8y + y^2) - 4\,dy + \pi\int_0^1 9 - 4\,dy$$

$$= \pi\int_1^2 (12 - 8y + y^2)\,dy + \pi\int_0^1 5\,dy$$

$$= \pi\left[12y - \frac{8y^2}{2} + \frac{y^3}{3}\right]_1^2 + \pi\left[5y\right]_0^1 = \boxed{\frac{22\pi}{3}}$$

7. **Set up but do not evaluate the integral** for the volume of the solid obtained by rotating the region bounded by $y = 2 - x^2$ and $y = 1$ about the line $y = -2$ **using the Washer/Disk Method**.



$r = 3$

$R = 4 - x^2$

$$V = \int_a^b \pi R^2 - \pi r^2\,dx$$

$$= \boxed{\int_{-1}^1 \pi(4-x^2)^2 - \pi(3)^2\,dx}$$

pts of intersection:

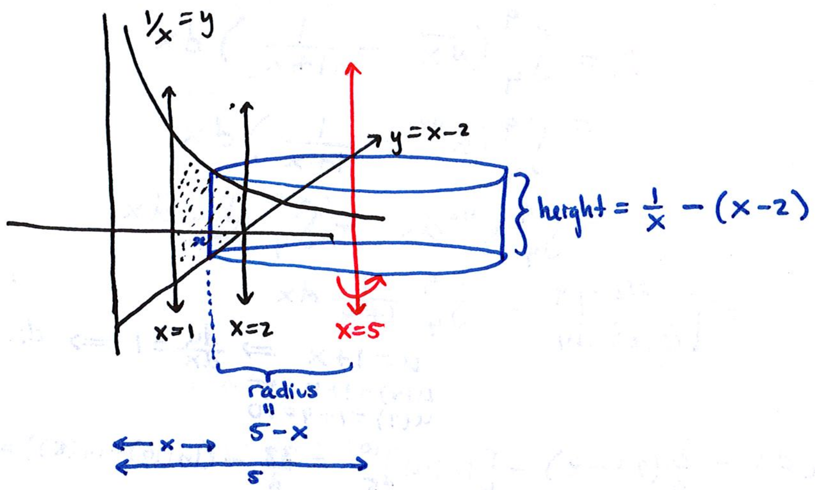
$2 - x^2 = 1$

$x^2 = 1$

$x = \pm\sqrt{1}$

$x = \pm 1$

8. **Set up but do not evaluate the integral** for the volume of the solid obtained by rotating the region bounded by $y = 1/x$, $y = x-2$, $x = 1$ and $x = 2$ about the line $x = 5$ **using the (cylindrical) Shell Method.**



radius $= 5-x$

height $= \frac{1}{x} - (x-2)$

$$V = \int_a^b 2\pi\,(\text{radius})(\text{height})\,dx$$

$$= \boxed{\int_1^2 2\pi\,(5-x)\left(\frac{1}{x} - (x-2)\right)dx}$$

9. **Set up but do not evaluate the integral** for the length of the curve $y = e^{\sin(x)}$ from $x = 1995$ to $x = 2013$.

$$\text{arc length} = \int_a^b \sqrt{1 + [y']^2}\,dx = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$$

$$y' = e^{\sin(x)} \cdot \frac{d}{dx}(\sin(x)) = e^{\sin(x)} \cdot \cos(x) = \cos(x)\,e^{\sin(x)}$$

$$\boxed{\text{arc length} = \int_{1995}^{2013} \sqrt{1 + \left[\cos(x)\,e^{\sin(x)}\right]^2}\,dx}$$

10. **Set up but do not evaluate the integral** for the surface area of the solid formed by rotating the portion of curve $y = \dfrac{\ln(x)}{2+x}$ from $x = 1776$ to $x = 2013$ about the x-axis.

$$\text{surface area} = \int_a^b 2\pi\,y\sqrt{1 + [y']^2}\,dx = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\,dx$$

$$y' = \frac{(2+x)\left(\frac{1}{x}\right) - \ln(x)(1)}{(2+x)^2}$$

$$\boxed{\text{surface area} = \int_{1776}^{2013} 2\pi\left(\frac{\ln(x)}{2+x}\right)\sqrt{1 + \left(\frac{(2+x)\left(\frac{1}{x}\right) - \ln(x)}{(2+x)^2}\right)^2}\,dx}$$