TEST 2

Math 152 - Calculus II

Score: 100 out of 100

10/11/2013

Name: Key

Read all of the following information before starting the exam:

- You have 50 minutes to complete the exam.
- Show all work, clearly and in order, if you want to get full credit. Please make sure you read the directions for each problem. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Please $\boxed{\text{box/circle}}$ or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 8 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. Evaluate $\int x \sin(5x)dx$.

LIATE

x $\sin(5x)$

$u = x \qquad dv = \sin(5x)$

$du = 1 \qquad v = -\frac{1}{5}\cos(5x)$

$$= -\frac{x}{5}\cos(5x) - \int\left(-\frac{1}{5}\cos(5x)\right)dx$$

$$= -\frac{x}{5}\cos(5x) + \frac{1}{5}\int \cos(5x)\,dx$$

$$= \boxed{-\frac{x}{5}\cos(5x) + \frac{1}{25}\sin(5x) + C}$$

2. Evaluate $\int \tan^{-1}(x)dx$.

LIATE

$\tan^{-1}(x)$ 1

$u = \tan^{-1}(x) \qquad dv = 1$

$du = \frac{1}{1+x^2} \qquad v = x$

$$= \tan^{-1}(x)\,x - \int \frac{x}{1+x^2}dx$$

$t = 1+x^2 \Rightarrow \frac{dt}{dx} = 2x \Rightarrow dx = \frac{dt}{2x}$

$$= \tan^{-1}(x)x - \int \frac{\cancel{x}}{t}\cdot\frac{dt}{2\cancel{x}}$$

$$= x\tan^{-1}(x) - \frac{1}{2}\int\frac{1}{t}\,dt = x\tan^{-1}(x) - \frac{1}{2}\ln|t| + C$$

$$= \boxed{x\tan^{-1}(x) - \frac{1}{2}\ln|1+x^2| + C}$$

3. Evaluate $\int \cos^2(3x)\sin^7(3x)dx$.

$$\int \cos^2(3x)\sin^6(3x)\sin(3x)dx = \int \cos^2(3x)\left(\sin^2(3x)\right)^3\sin(3x)dx$$

$$= \int \cos^2(3x)\left(1-\cos^2(3x)\right)^3\sin(3x)\,dx$$

$u = \cos(3x) \Rightarrow \frac{du}{dx} = -3\sin(3x) \Rightarrow dx = \frac{du}{-3\sin(3x)}$

$$= \int u^2(1-u^2)^3\,\cancel{\sin(3x)}\cdot\frac{du}{-3\cancel{\sin(3x)}} = -\frac{1}{3}\int u^2(1-u^2)^3\,du$$

$(1-u^2)(1-u^2)(1-u^2) = (1-u^2)(1-2u^2+u^4)$

$= (1-2u^2+u^4-u^2+2u^4-u^6)$

$$= -\frac{1}{3}\int u^2(1-3u^2+3u^4-u^6)du$$

$$= -\frac{1}{3}\int(u^2-3u^4+3u^6-u^8)du$$

$$= -\frac{1}{3}\left[\frac{u^3}{3}-\frac{3u^5}{5}+\frac{3u^7}{7}-\frac{u^9}{9}\right]+C$$

$$\boxed{-\frac{1}{3}\left[\frac{\cos^3(3x)}{3} - \frac{3\cos^5(3x)}{5} + \frac{3\cos^7(3x)}{7} - \frac{\cos^9(3x)}{9}\right]+C}$$

4. Evaluate $\int \frac{1}{x^2\sqrt{x^2+25}}dx$.

$$x = 5\tan\theta$$

$$\frac{dx}{d\theta} = 5\sec^2\theta \Rightarrow dx = 5\sec^2\theta\, d\theta$$

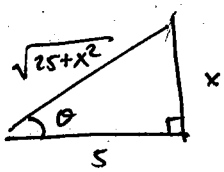
$$\sqrt{x^2+25} = \sqrt{(5\tan\theta)^2+25} = \sqrt{25\tan^2\theta+25} = \sqrt{25(\tan^2\theta+1)} = \sqrt{25\sec^2\theta} = 5\sec\theta$$

$$\int \frac{1}{(5\tan\theta)^2\,\cancel{5\sec\theta}}\,\cancel{5}\sec^2\theta\, d\theta$$

$$= \frac{1}{25}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta = \frac{1}{25}\int \frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}\,d\theta = \frac{1}{25}\int \frac{\cos^2\theta}{\sin^2\theta\cos\theta}\,d\theta = \frac{1}{25}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

$$u = \sin\theta \Rightarrow \frac{du}{d\theta} = \cos\theta$$

$$= \frac{1}{25}\int \frac{1}{u^2}\cdot du = \frac{1}{25}\int u^{-2}\,du = \frac{1}{25}\left[\frac{u^{-1}}{-1}\right] + C = -\frac{1}{25}\left[\frac{1}{u}\right] + C = -\frac{1}{25}\left(\frac{1}{\sin\theta}\right) + C$$



$x = 5\tan\theta$

$\tan\theta = \frac{x}{5} = \frac{opp}{adj}$

$\sin\theta = \frac{opp}{hyp} = \frac{x}{\sqrt{25+x^2}}$

$$= -\frac{1}{25}\left(\frac{1}{\frac{x}{\sqrt{25+x^2}}}\right) + C$$

$$= \boxed{-\frac{1}{25}\left(\frac{\sqrt{25+x^2}}{x}\right) + C}$$

5. Evaluate $\int \frac{4x-1}{x^2+3x-10}dx$.

$$\frac{4x-1}{x^2+3x-10} = \frac{4x-1}{(x+5)(x-2)} = \frac{A}{x+5} + \frac{B}{x-2}$$

$$= \frac{A(x-2)+B(x+5)}{(x+5)(x-2)}$$

$$4x-1 = Ax - 2A + Bx + 5B$$

$$4x-1 = (A+B)x - 2A + 5B$$

$4 = A+B \qquad -1 = -2A+5B$

$A = 4-B \longrightarrow -1 = -2(4-B)+5B$

$-1 = -8 + 2B + 5B$

$7 = 7B$

$A = 4-1 \longleftarrow \underline{B = 1}$

$\underline{A = 3}$

$$\int \frac{4x-1}{x^3+3x-10}dx = \int\left(\frac{3}{x+5} + \frac{1}{x-2}\right)dx = \boxed{3\ln|x+5| + 1\ln|x-2| + C}$$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

6. Evaluate $\int \sec^6(2x)\tan^3(2x)dx$.

SOL 1:

$\int \sec^2(2x)\sec^4(2x)\tan^3(2x)\,dx$

$\int \sec^2(2x)\left(\tan^2(2x)+1\right)^2 \tan^3(2x)\,dx$

$u = \tan(2x) \Rightarrow \frac{du}{dx} = 2\sec^2(2x)$

$\Rightarrow dx = \frac{du}{2\sec^2(2x)}$

$= \frac{1}{2}\int (u^2+1)^2 u^3\,du$

$= \frac{1}{2}\int (u^4+2u^2+1)u^3\,du$

$= \frac{1}{2}\int u^7 + 2u^5 + u^3\,du$

$\frac{1}{2}\left[\frac{u^8}{8} + \frac{2u^6}{6} + \frac{u^4}{4}\right] + C = \boxed{\frac{1}{2}\left[\frac{\tan^8(2x)}{8} + \frac{\tan^6(2x)}{3} + \frac{\tan^4(2x)}{4}\right] + C}$

SOL 2:

$\int \sec^5(2x)\tan^2(2x)\tan(2x)\sec(2x)\,dx$

$\int \sec^5(2x)\left(\sec^2(2x)-1\right)\tan(2x)\sec(2x)\,dx$

$u = \sec(2x) \Rightarrow \frac{du}{dx} = 2\sec(2x)\tan(2x)$

$\Rightarrow dx = \frac{du}{2\sec(2x)\tan(2x)}$

$\frac{1}{2}\int u^5(u^2-1)\,du = \frac{1}{2}\int u^7 - u^5\,du$

$= \frac{1}{2}\left[\frac{u^8}{8} - \frac{u^6}{6}\right] + C$

$= \boxed{\frac{1}{2}\left[\frac{\sec^8(2x)}{8} - \frac{\sec^6(2x)}{6}\right] + C}$

7. Use polynomial long division to evaluate $\int \frac{x^4+8}{x-3}dx$.

$$\begin{array}{r} x^3 + 3x^2 + 9x + 27 \\ x-3 \,\big)\, x^4 \qquad\qquad\quad 8 \\ -(x^4 - 3x^3) \\ 3x^3 \qquad\qquad \\ -(3x^3 - 9x^2) \\ 9x^2 + \qquad \\ -(9x^2 - 27x) \\ 27x + 8 \\ -(27x - 81) \\ 89 \end{array}$$

$= \int x^3 + 3x^2 + 9x + 27 + \frac{89}{x-3}\,dx$

$= \boxed{\frac{x^4}{4} + \frac{3x^3}{3} + \frac{9x^2}{2} + 27x + 89\ln|x-3| + C}$

$= \boxed{\frac{x^4}{4} + x^3 + \frac{9x^2}{2} + 27x + 89\ln|x-3| + C}$

8. Write out the FORM of the partial fraction decomposition for the following (DO NOT find the numerical values for the unknown coefficients).

(a) $\frac{x^2-3x+10}{x^3(x-4)(x+3)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-4} + \frac{E}{x+3} + \frac{F}{(x+3)^2}$

(b) $\frac{2x-20}{x^3+x^2} = \frac{2x-20}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

(c) $\frac{2x^3+4x-15}{(x-2)(x^2-4)^2(x^2+9)^2} = \frac{2x^3+4x-15}{(x-2)(x+2)^2(x-2)^2(x^2+9)^2} = \frac{2x^3+4x-15}{(x+2)^2(x-2)^3(x^2+9)^2}$

$= \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x-2} + \frac{D}{(x-2)^2} + \frac{E}{(x-2)^3} + \frac{Fx+G}{x^2+9} + \frac{Hx+I}{(x^2+9)^2}$