TEST 3

Math 152 - Calculus II | Score: ________ out of 100

11/8/2013 | Name: Key

Read all of the following information before starting the exam:

- You have 50 minutes to complete the exam.
- Show all work, clearly and in order, if you want to get full credit. Please make sure you read the directions for each problem. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Please $\boxed{\text{box/circle}}$ or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 8 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. Determine whether the sequence converges, and if so find its limit.

(a) $\left\{\dfrac{5n^6-n^2+1}{3n^6-n^3+1}\right\}_{n=1}^{\infty}$

$$\lim_{n\to\infty}\frac{5n^6-n^2+1}{3n^6-n^3+1}\cdot\frac{(1/n^6)}{(1/n^6)}=\lim_{n\to\infty}\frac{5-\frac{1}{n^4}+\frac{1}{n^6}}{3-\frac{1}{n^3}+\frac{1}{n^6}}=\frac{5-0+0}{3-0+0}=\boxed{\frac{5}{3}}$$

$\boxed{\text{Converges}}$

(b) $\left\{\dfrac{2n^2}{3\ln(n)}\right\}_{n=1}^{\infty}$

$$\lim_{x\to\infty}\frac{2x^2}{3\ln(x)}\overset{L'H}{=}\lim_{x\to\infty}\frac{4x}{3\left(\frac{1}{x}\right)}=\lim_{x\to\infty}\frac{4x^2}{3}=\infty \quad \boxed{\text{diverges}}$$

(c) $\left\{\dfrac{\cos(3n)}{n^5}\right\}_{n=1}^{\infty}$

$$\frac{-1}{n^5}\le\frac{\cos(3n)}{n^5}\le\frac{1}{n^5}$$

Also, $\left.\begin{array}{l}\displaystyle\lim_{n\to\infty}-\frac{1}{n^5}=0\\ \displaystyle\lim_{n\to\infty}\frac{1}{n^5}=0\end{array}\right\}$ By squeeze thm $\displaystyle\lim_{n\to\infty}\frac{\cos(3n)}{n^5}=\boxed{0}$

$\boxed{\text{Converges}}$

2. Show that the given sequence is strictly increasing or strictly decreasing. (Note: You may not just write out the first few terms of the sequence to answer this question.)

$$\left\{\frac{\ln(n+5)}{n+5}\right\}_{n=1}^{\infty}$$

3 Methods! Best method: $f(x)=\dfrac{\ln(x+5)}{x+5}$

$$f'(x)=\frac{\frac{x+5}{x+5}-\ln(x+5)}{(x+5)^2}=\frac{1-\ln(x+5)}{(x+5)^2}<0$$

$\ominus$ $\oplus$

The sequence is strictly decreasing

3. Each series below is geometric. Determine both a and r. Then decide whether the series converges or diverges. If the series converges, then find its sum. If it diverges, write "NO SUM."

(a) $\sum_{k=0}^{\infty}(-1)^k\frac{3^{2k+1}}{8^k}$

SOL 1

$$= (-1)^0\frac{3^{0+1}}{8^0} + (-1)^1\frac{3^{2+1}}{8^1} + \cdots$$

$$= \frac{1\cdot 3^1}{1} + \frac{(-1)3^3}{8} + \cdots$$

$$= \underset{a}{3} + 3\underset{r}{\left(-\tfrac{9}{8}\right)} + \cdots$$

$a = 3$

$r = -9/8$

sum= NO SUM.

$|r| = 9/8 \geq 1$. So diverges.

SOL 2: $\sum_{k=0}^{\infty}(-1)^k\frac{3^{2k+1}}{8^k} = \sum_{k=0}^{\infty}(-1)^k\frac{3^{2k}\cdot 3^1}{8^k}$

$$= \sum_{k=0}^{\infty} 3\cdot\frac{(-1)^k 9^k}{8^k}$$

$$= \sum_{k=0}^{\infty} \underset{a}{3}\underset{r}{\left(-\tfrac{9}{8}\right)}^k$$

(b) $\sum_{k=1}^{\infty}3(-5)^{k-1}$

or you can write this sort like above.

$a = 3$

$r = -5$

sum= NO SUM

$|r| = 5 \geq 1$, diverges.

4. Use the **Divergence Test** on each of the following to determine whether the given series diverges. If the test yields no conclusion, then be sure to say so. You must set up, evaluate, and interpret the correct limit to earn credit.

(a) $\sum_{n=1}^{\infty}\tan^{-1}(n)$

$\lim_{n\to\infty}\tan^{-1}(n) = \frac{\pi}{2} \neq 0$ so the series $\boxed{\text{diverges}}$

(b) $\sum_{n=1}^{\infty}\frac{n}{4n^3-1}$

$$\lim_{n\to\infty}\frac{n}{4n^3-1} = \lim_{n\to\infty}\frac{n}{4n^3-1}\frac{(1/n^3)}{(1/n^3)} = \lim_{n\to\infty}\frac{1/n^2}{4-1/n^3} = \frac{0}{4-0} = 0$$

$\boxed{\text{NO INFO}}$

5. Use the **Integral Test** to determine whether the given series converges or diverges. Clearly identify the function $f(x)$ you are embedding the sequence of terms into. You may assume that $f(x)$ is positive, decreasing and continuous for $x \geq 1$, so you do not need to verify this. Just use the integral test and state your conclusion.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+5}}$$

$$\int_1^{\infty} \frac{1}{\sqrt{x+5}}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{\sqrt{x+5}}\,dx = \lim_{t\to\infty} \int_1^t (x+5)^{-1/2}\,dx$$

$u = x+5 \rightarrow \frac{du}{dx} = 1 \rightarrow du = dx$

$u(1) = 6$

$u(t) = t+5$

$$= \lim_{t\to\infty} \int_6^{t+5} u^{-1/2}\,du$$

$$= \lim_{t\to\infty} \left[2u^{1/2}\right]_6^{t+5}$$

$$= \lim_{t\to\infty} \left[2\sqrt{t+5} - 2\sqrt{6}\right]$$

$$= \infty$$

diverges $\rightarrow$ original series $\boxed{\text{diverges}}$

6. Use the **Comparison Test** to determine whether the given series converges or diverges. Clearly write down an inequality, determine the convergence or divergence of your comparison series $\sum_{n=1}^{\infty} b_n$, and then write your conclusion.

$$\sum_{n=1}^{\infty} \frac{\sin^2(n)}{3^n}$$

$$\frac{\sin^2(n)}{3^n} \leq \frac{1}{3^n}$$

Also, $\sum_{n=1}^{\infty} \frac{1}{3^n} = \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$

converges (geometric $|r| = 1/3 < 1$)

Hence, the original series also $\boxed{\text{converges}}$

7. Use the **Limit Comparison Test** to determine whether the given series converges or diverges. Clearly write down what a_n and b_n are, compute the appropriate limit, determine the convergence or divergence of your comparison series $\sum_{n=1}^{\infty} b_n$, and then write your conclusion.

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^4+1}}$$

$$a_n = \frac{n}{\sqrt{n^4+1}} \qquad b_n = \frac{n}{\sqrt{n^4}} = \frac{n}{n^2} = \frac{1}{n}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n}{\sqrt{n^4+1}}}{\frac{n}{\sqrt{n^4}}} = \lim_{n\to\infty} \frac{\sqrt{n^4}}{\sqrt{n^4+1}} = \lim_{n\to\infty} \sqrt{\frac{n^4}{n^4+1}}$$

$$= \sqrt{1} = 1$$

Also, $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges (harmonic series)

Hence, original series also $\boxed{\text{diverges}}$

SOLVE ONE OF THE FOLLOWING:

8. **Please indicate which problem you want me to grade by filling a single box below, otherwise I will grade the first problem worked on:**

Determine whether the series converges or diverges. You may use **any method (state the method you are using)** that will give you a conclusion!

(a) $\sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{n^2+1}$ ☐ **I will solve this one**

(b) $\sum_{n=1}^{\infty} \frac{4}{n^5+5^n}$ ☐ **I will solve this one**

(a) Sol 1: integral test...

Sol 2: comparison test:

$$\frac{\tan^{-1}(n)}{n^2+1} \leq \frac{\pi/2}{n^2+1} \leq \frac{\pi/2}{n^2}$$

Also $\sum_{n=1}^{\infty} \frac{\pi/2}{n^2} = \frac{\pi}{2}\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p-series $p=2>1$)

So original series $\boxed{\text{converges}}$

(b) Sol 1: comparison test:

$$\frac{4}{n^5+5^n} < \frac{4}{n^5}$$

Also, $\sum_{n=1}^{\infty} \frac{4}{n^5} = 4\sum_{n=1}^{\infty} \frac{1}{n^5}$ converges (p-series $p=5>1$)

So original series $\boxed{\text{converges}}$

Sol 2: Another comparison....