MACLAURIN SERIES, DIFFERENTIATING AND INTEGRATING POWER SERIES

1. Important Maclaurin Series

(1) $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots,$ where $-1 < x < 1$.

(2) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots,$ where $-\infty < x < \infty$.

(3) $\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots,$ where $-\infty < x < \infty$.

(4) $\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots,$ where $-\infty < x < \infty$.

(5) $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n = 1 - x + x^2 - x^3 + \cdots,$ where $-1 < x < 1$.

(6) $\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n} = 1 - x^2 + x^4 - x^6 + \cdots,$ where $-1 < x < 1$.

(7) $\tan^{-1}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots,$ where $-1 \leq x \leq 1$.

(8) $\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots,$ where $-1 < x \leq 1$.

2. Differentiating and Integrating Power Series

Let

$$f(x) = \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 \cdots .,$$

for some interval of convergence with $R > 0$. Then we can differentiate and integrate the power series term by term (over the interior of the interval of convergence):

(1) $f'(x) = c_1 + 2c_2 x + 3c_3 x^2 + \cdots = \sum_{n=0}^{\infty} n c_n x^{n-1}.$

(2) $\int f(x)dx = C + c_0 x + \frac{c_1 x^2}{2} + \frac{c_3 x^3}{3} + \cdots = C + \sum_{n=0}^{\infty} \frac{c_n x^{n+1}}{n+1}.$

and the radius of convergence is R for both 1 and 2.

Note: you can do the same for general power series centered at x_0.

Nathan Reff, Department of Mathematics, Alfred University, Alfred, NY 14802, U.S.A.
E-mail address: reff@alfred.edu