

Name: _____

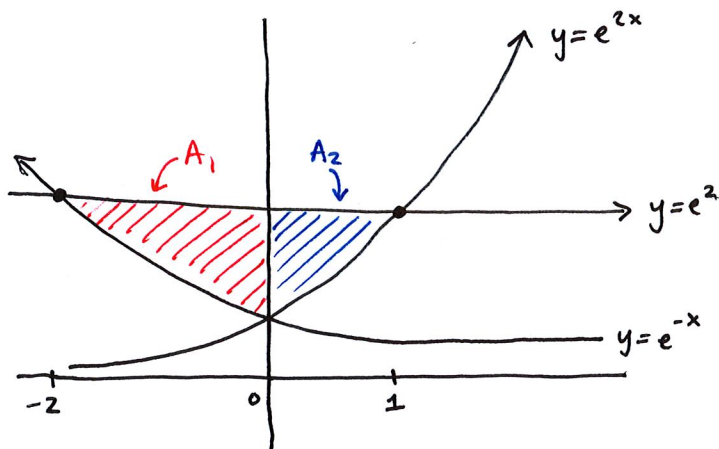
Key

Show all work clearly and in order. Please box your answers.

SOLVE ONE OF THE FOLLOWING:

Please indicate which problem you do NOT want me to grade by putting a GIANT X through it, otherwise I will grade the first problem worked on:

1. Find the area of the region bounded by $y = e^{2x}$, $y = e^{-x}$ and $y = e^2$. Note: you must also draw a clear picture to receive full credit.

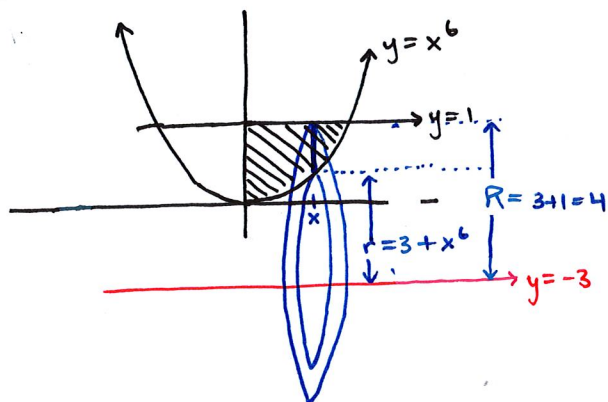


points of intersection: $e^2 = e^{-x} \quad e^2 = e^{2x}$
 $2 = -x \quad 2 = 2x$
 $x = -2 \quad x = 1$

$$\text{Area} = A_1 + A_2$$

$$\begin{aligned} &= \int_{-2}^0 (e^2 - e^{-x}) dx + \int_0^1 (e^2 - e^{2x}) dx \\ &= [e^2 x + e^{-x}]_{-2}^0 + [e^2 x - \frac{e^{2x}}{2}]_0^1 \\ &= [(e^2)(0) + e^0] - (e^2(-2) + e^2) + [(e^2)(1) - \frac{e^2}{2}] - (0 - \frac{1}{2}) \\ &= [e^2 + 1] + [\frac{e^2}{2} + \frac{1}{2}] \\ &= \boxed{\frac{3}{2}e^2 + \frac{3}{2}} \approx 12.5836 \end{aligned}$$

2. Set up but do not evaluate the integral for the volume of the solid obtained by rotating the region bounded by $y = x^6$, $y = 1$ and the y -axis with $x \geq 0$ (so the region is in the first quadrant) about the line $y = -3$. Note: you must also draw a clear picture to receive full credit.

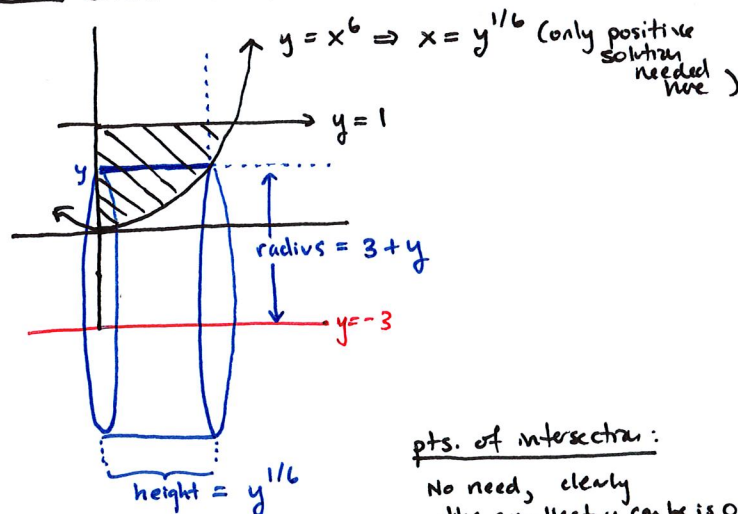
Sol 1: (Washer/Disk Method)

$r = \text{inner radius} = 3 + x^6$
 $R = \text{outer radius} = 3 + 1 = 4$

pts. of intersection

$1 = x^6$
 $\pm 1^{1/6} = x$
 $x = \pm 1$
 Only $x = +1$ is in the right region)

$$V = \int_0^1 [\pi(4)^2 - \pi(3+x^6)^2] dx$$

Sol 2: (Shell Method)

height = $y^{1/6}$
 radius = $3 + y$

pts. of intersection:

No need, clearly the smallest y can be is 0 and the largest y can be is 1. ($y = 1$ is given)

$$V = \int_0^1 2\pi(3+y)(y^{1/6}) dy$$