

Name: key

Show all work clearly and in order. Please box your answers.

1. Set up but do not evaluate the integral for the exact arc length of the curve $y = \sin^2(3x) + 4$ from $x = \pi/2$ to $x = \pi$.

$$\begin{aligned} y &= \sin^2(3x) + 4 = (\sin(3x))^2 + 4 \\ y' &= 2 \sin(3x) \cdot \frac{d}{dx}(\sin(3x)) \\ &= 2 \sin(3x) \cos(3x) \frac{d}{dx}(3x) \\ &= 2 \sin(3x) \cos(3x) \cdot 3 \\ &= 6 \sin(3x) \cos(3x) \end{aligned}$$

$$\text{Length} = \int_a^b \sqrt{1 + [y']^2} dx$$

$$\text{Length} = \boxed{\int_{\pi/2}^{\pi} \sqrt{1 + [6 \sin(3x) \cos(3x)]^2} dx}$$

2. Set up but do not evaluate the integral for the area of the surface generated by revolving $y = \sqrt{4 - x^2}$ about the x -axis from $x = -1$ to $x = 1$.

$$\begin{aligned} y &= (4 - x^2)^{1/2} \\ y' &= \frac{1}{2} (4 - x^2)^{-1/2} \frac{d}{dx}(4 - x^2) \\ &= \frac{1}{2} (4 - x^2)^{-1/2} (-2x) \end{aligned}$$

$$\text{surface area} = \int_a^b 2\pi y \sqrt{1 + [y']^2} dx$$

$$= \boxed{\int_{-1}^1 2\pi \sqrt{4 - x^2} \sqrt{1 + \left[\frac{1}{2} (4 - x^2)^{-1/2} (-2x)\right]^2} dx}$$