Math 152 - Fall 2013 - Quiz #4 Score: ________ out of 10.

Name: Key

Show all work clearly and in order. Please box your answers.

1. $\int \sec(x)dx$.

A. $-\ln|\cos(x)| + C$

B. $\ln|\sec(x)| + C$

C. $\tan(x) + C$

$\boxed{\text{D. } \ln|\sec(x) + \tan(x)| + C}$

2. $\int \sec^2(x)dx$.

A. $\ln|\sec(x) + \tan(x)| + C$

$\boxed{\text{B. } \tan(x) + C}$

C. $-\ln|\cos(x)| + C$

D. $\ln|\sec(x)| + C$

SOLVE 2 OF THE FOLLOWING INTEGRALS:

Please indicate which integrals you do NOT want me to grade by putting a GIANT X through them, otherwise I will grade the two integrals worked on:

3. Evaluate $\int x^2 e^{3x} dx$.

LIATE

x^2 e^{3x}

$u = x^2$ $dv = e^{3x}$

$du = 2x$ $v = \frac{1}{3}e^{3x}$

$x^2\left(\frac{1}{3}e^{3x}\right) - \int \frac{1}{3}e^{3x} 2x dx$

$= \frac{x^2 e^{3x}}{3} - \frac{2}{3}\int x e^{3x} dx = \frac{1}{3}x^2e^{3x} - \frac{2}{3}\left(x\left(\frac{1}{3}\right)e^{3x} - \int \frac{1}{3}e^{3x}dx\right)$

LIATE

x e^{3x}

$u = x$ $dv = e^{3x}$

$du = 1$ $v = \frac{1}{3}e^{3x}$

$= \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{9}\left(\frac{1}{3}\right)e^{3x} + C$

$= \boxed{\frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C}$

4. Evaluate $\int \sin^2(x)\cos^3(x)dx$.

$= \int \sin^2(x)\cos^2(x)\cos(x)dx$

$= \int \sin^2(x)(1-\sin^2(x))\cos(x)dx = \int u^2(1-u^2)\cancel{\cos(x)}\frac{du}{\cancel{\cos(x)}}$

$u = \sin(x)$

$\frac{du}{dx} = \cos(x) \Rightarrow dx = \frac{du}{\cos(x)}$

$= \int u^2 - u^4 du = \frac{u^3}{3} - \frac{u^5}{5} + C$

$= \boxed{\frac{\sin^3(x)}{3} - \frac{\sin^5(x)}{5} + C}$

5. Evaluate $\int \sin^{-1}(x)dx$.

LIATE

$\sin^{-1}(x)$ 1

$u = \sin^{-1}(x)$ $dv = 1\,dx$

$du = \frac{1}{\sqrt{1-x^2}}$ $v = x$

$x\sin^{-1}(x) - \int x\left(\frac{1}{\sqrt{1-x^2}}\right)dx = x\sin^{-1}(x) - \int \frac{x}{\sqrt{1-x^2}}dx$

$u = 1-x^2 \Rightarrow \frac{du}{dx} = -2x \Rightarrow dx = \frac{du}{-2x}$

$= x\sin^{-1}(x) - \int \frac{x}{\sqrt{u}} \cdot \frac{du}{-2x}$

$= x\sin^{-1}(x) + \frac{1}{2}\int u^{-1/2}du$

$= x\sin^{-1}(x) + \frac{1}{2}\frac{u^{1/2}}{(1/2)} + C = \boxed{x\sin^{-1}(x) + (1-x^2)^{1/2} + C}$

6. Evaluate $\int \sec^2(4x)\tan^3(4x)dx$.

SOL 1: $u = \tan(4x)$

$\frac{du}{dx} = 4\sec^2(4x) \Rightarrow dx = \frac{du}{4\sec^2(4x)}$

$\int \sec^2(4x)\tan^3(4x)dx = \int \cancel{\sec^2(4x)}\, u^3 \cdot \frac{du}{(4)\cancel{\sec^2(4x)}}$

$= \frac{1}{4}\int u^3 du = \frac{u^4}{4\cdot 4} + C = \boxed{\frac{1}{16}\tan^4(4x) + C}$

SOL 2: $\int \sec^2(4x)\tan^3(4x)dx = \int \sec^2(4x)\tan^2(4x)\tan(4x)dx = \int \sec(4x)\tan^2(4x)\sec(4x)\tan(4x)dx$

$u = \sec(4x) \Rightarrow \frac{du}{dx} = 4\sec(4x)\tan(4x)$

$\Rightarrow dx = \frac{du}{4\sec(4x)\tan(4x)}$

so $\int \sec(4x)(\sec^2(4x)-1)\sec(4x)\tan(4x)dx = \int u(u^2-1)\frac{du}{4} = \frac{1}{4}\int u^3 - u\,du = \frac{1}{4}\left(\frac{u^4}{4} - \frac{u^2}{2}\right) + C$

$$= \boxed{\frac{\sec^4(4x)}{16} - \frac{\sec^2(4x)}{2} + C}$$