Math 152 - Fall 2013 - Quiz #6 **Score:** ________ out of 10.

Name: Key

Show all work clearly and in order. Please box your answers.

1. For each of the following improper integrals determine whether they converge or diverge.

(a) $\displaystyle\int_5^\infty \frac{1}{x\ln(x)}dx.$

$$= \lim_{t\to\infty} \int_5^t \frac{1}{x\ln(x)}\,dx$$

$u = \ln(x) \implies \frac{du}{dx} = \frac{1}{x} \implies dx = x\,du$

$u(5) = \ln(5)$
$u(t) = \ln(t)$

$$= \lim_{t\to\infty} \int_{\ln(5)}^{\ln(t)} \frac{1}{\cancel{x} u}\cdot \cancel{x}\,du$$

$$= \lim_{t\to\infty} \int_{\ln(5)}^{\ln(t)} \frac{1}{u}\,du = \lim_{t\to\infty} \Big[\ln|u|\Big]_{\ln(5)}^{\ln(t)}$$

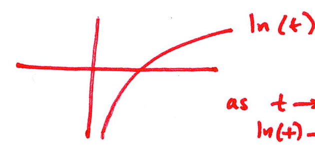
$$= \lim_{t\to\infty} \Big[\ln|\ln(t)| - \ln|\ln(5)|\Big]$$

$$= \infty$$

$\boxed{\text{diverges}}$

Why?

Note:



as $t\to\infty$
$\ln(t)\to\infty$
so
$\ln(\ln(t))\to\infty$
since this $\to\infty$ (above).

(b) Evaluate $\displaystyle\int_4^9 \frac{1}{\sqrt{x-4}}dx.$

$4 \leftarrow t$ $\quad 9$

this function is undefined at $x=4$ (in fact $x \le 4$) so it (the integral.) is improper.

$$= \lim_{t\to 4^+} \int_t^9 \frac{1}{\sqrt{x-4}}\,dx$$

$u = x-4 \implies \frac{du}{dx} = 1 \implies du = dx$

$u(t) = t-4$
$u(9) = 9-4 = 5$

$$= \lim_{t\to 4^+} \int_{t-4}^{5} \frac{1}{\sqrt{u}}\,du$$

$$= \lim_{t\to 4^+} \int_{t-4}^{5} u^{-1/2}\,du = \lim_{t\to 4^+} \left[\frac{u^{1/2}}{1/2}\right]_{t-4}^{5} = \lim_{t\to 4^+} \Big[2\sqrt{u}\Big]_t^5$$

$$= \lim_{t\to 4^+} \left(2\sqrt{5} - 2\sqrt{t-4}\right)$$

$$= 2\sqrt{5} - 2\sqrt{0}$$

$$= 2\sqrt{5} - 0$$

$$= \boxed{2\sqrt{5}} \text{ so } \boxed{\text{converges}}$$