Math 152 - Fall 2013 - Quiz #7 **Score:** ________ out of 10.

Name: Key

Show all work clearly and in order. Please box your answers.

# SOLVE ONE OF THE FOLLOWING:

**You must do all parts of a problem that you choose. Please indicate which problem you do NOT want me to grade by putting a GIANT X through it, otherwise I will grade the first problem worked on:**

1. Determine whether the sequence converges, and if so find its limit.

(a) $\left\{\frac{\ln(n)}{4n}\right\}_{n=2}^{\infty}$ Let $f(x) = \frac{\ln(x)}{4x}$ (embed the sequence)

$$\lim_{x\to\infty} \frac{\overbrace{\ln(x)}^{\text{goes to } \infty}}{\underbrace{4x}_{\text{goes to } \infty}} \overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{d}{dx}(\ln(x))}{\frac{d}{dx}(4x)} = \lim_{x\to\infty} \frac{\left(\frac{1}{x}\right)}{4} = \lim_{x\to\infty} \frac{1}{4x} = 0.$$

$\boxed{\text{Sequence converges}}$

(b) $\left\{\frac{\sin(2n+1)}{n}\right\}_{n=1}^{\infty}$ since $-1 \le \sin(2n+1) \le 1$ we can write

$$\frac{-1}{n} \le \frac{\sin(2n+1)}{n} \le \frac{1}{n}$$

Also, Since $\lim_{n\to\infty} -\frac{1}{n} = 0$ AND $\lim_{n\to\infty} \frac{1}{n} = 0$

by squeeze thm $\lim_{n\to\infty} \frac{\sin(2n+1)}{n} = 0$. $\boxed{\text{Sequence converges}}$

2. Show that the given sequence is strictly increasing or strictly decreasing.

$$\left\{\frac{2n}{7n-1}\right\}_{n=1}^{\infty}$$

SOL 1: Let $f(x) = \frac{2x}{7x-1} \Rightarrow f'(x) = \frac{(7x-1)2 - 2x(7)}{(7x-1)^2} = \frac{14x - 2 - 14x}{(7x-1)^2} = \frac{\overbrace{-2}^{\ominus}}{\underbrace{(7x-1)^2}_{\oplus}} < 0$

so the sequence is strictly decreasing.

SOL 2: Show $a_{n+1} - a_n < 0$ ... (work not shown)

SOL 3: Show $\frac{a_{n+1}}{a_n} < 1$ .... (work not shown).

**The sequence is strictly** decreasing.

3. Each series below is geometric. Determine both $a$ and $r$. Then decide whether the series converges or diverges. If the series converges, then find its sum. If it diverges, write "NO SUM."

(a) $\sum_{k=1}^{\infty}\left(-\frac{1}{4}\right)^{k-1}$

$a =$ 1

$r =$ $-1/4$

sum= $4/5$

SOL 1 $\sum_{k=1}^{\infty}(-1/4)^{k-1}$ is already in the proper form.

$a=1$ $r=-1/4$

Since $|r| = |-1/4| = 1/4 < 1$

series converges to $\frac{a}{1-r} = \frac{1}{1-(-1/4)} = \frac{1}{5/4} = 4/5$

SOL 2: expand the sum.

$$\sum_{k=1}^{\infty}(-1/4)^{k-1} = (-1/4)^0 + (-1/4)^1 + (-1/4)^2 + \cdots$$

$$= \underbrace{1 - 1/4 + 1/4^2 + \cdots}_{\text{geometric!}}$$

$a =$ first term $= 1$

$r =$ ratio of consecutive terms $= \frac{(-1/4)}{1} = -1/4.$

Same conclusions as SOL 1.

(b) $\sum_{k=0}^{\infty}(-1)^k\frac{2^{3k}}{5^{k+1}}$

$a =$ $1/5$

$r =$ $-8/5$

sum= NO SUM.

SOL 1: $\sum_{k=0}^{\infty}(-1)^k\frac{2^{3k}}{5^{k+1}}$

starts at 0, want powers of k.

$$= \sum_{k=0}^{\infty}\frac{(-1)^k(2^3)^k}{5^k \cdot 5}$$

$$= \sum_{k=0}^{\infty}\left(\frac{1}{5}\right)\left(\frac{(-1)^k(8)^k}{5^k}\right)$$

$$= \sum_{k=0}^{\infty}\left(\frac{1}{5}\right)\left(-\frac{8}{5}\right)^k$$

$a = 1/5$ $r = -8/5$

$|r| = 8/5 \geq 1$ diverges.

SOL 2: $\sum_{k=0}^{\infty}(-1)^k\frac{2^{3k}}{5^{k+1}} = (-1)^0\frac{2^0}{5^1} + \frac{(-1)^1 2^3}{5^2} + \frac{(-1)^2 2^6}{5^3} + \cdots$

$$= \underbrace{\frac{1}{5} - \frac{2^3}{5^2} + \frac{2^6}{5^3} + \cdots}_{\text{geometric.}}$$

$a = 1/5$

$r = (-2^3/5^2)/(1/5) = \frac{-8}{5}$

Same conclusions as SOL. 1.