Math 152 - Fall 2013 - Quiz #8 **Score:** ________ out of 10.

Name: Key

Show all work clearly and in order. Please box your answers.

SOLVE ONE OF THE FOLLOWING:

Please indicate which problems you do NOT want me to grade by putting a GIANT X through thm, otherwise I will grade the first problem worked on:

1. Use the **Integral Test** to determine whether the given series converges or diverges. Clearly identify the function $f(x)$ you are embedding the sequence of terms into. You may assume that $f(x)$ is positive, decreasing and continuous for $x \geq 1$, so you do not need to verify this. Just use the integral test and state your conclusion.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n+10}}$$

$f(x) = \frac{1}{(x+10)^{1/5}}$

$u = x+10 \Rightarrow \frac{du}{dx} = 1 \Rightarrow dx = du$

$u(1) = 11$

$u(t) = t+10$

$$\int_1^{\infty} \frac{1}{(x+10)^{1/5}}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{(x+10)^{1/5}}\,dx = \lim_{t\to\infty} \int_{11}^{t+10} \frac{1}{u^{1/5}}\,du = \lim_{t\to\infty} \int_{11}^{t+10} u^{-1/5}\,du$$

$$= \lim_{t\to\infty} \left[\frac{5u^{4/5}}{4}\right]_{11}^{t+10} = \lim_{t\to\infty} \frac{5}{4}\left[(t+10)^{4/5} - 11^{4/5}\right] = \infty$$

diverges. so the series also $\boxed{\text{diverges}}$

2. Use the **Comparison Test** to determine if the series $\sum_{n=1}^{\infty} \frac{\cos^4(2n)}{n^7}$ converges or diverges.

$$\frac{\cos^4(2n)}{n^7} \leq \frac{1}{n^7}$$

Also, $\sum_{n=1}^{\infty} \frac{1}{n^7}$ converges (p-series $p = 7 > 1$).

so $\sum_{n=1}^{\infty} \frac{\cos^4(2n)}{n^7}$ also $\boxed{\text{converges}}$

3. Use the **Limit Comparison Test** to determine if the series $\sum_{n=1}^{\infty} \frac{2n^2 - n - 1}{3n^5 + 3n - 3}$ converges or diverges.

$a_n = \frac{2n^2 - n - 1}{3n^5 + 3n - 3}$ $\qquad \frac{2n^2}{3n^5} = b_n$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \left(\frac{2n^2 - n - 1}{3n^5 + 3n - 3}\right)\cdot\left(\frac{3n^5}{2n^2}\right) = \lim_{n\to\infty} \left(\frac{6n^7 - 3n^6 - 3n^5}{6n^7 + 6n^3 - 6n^2}\right)\frac{(1/n^7)}{(1/n^7)}$$

$$= \lim_{n\to\infty} \frac{6 - 3/n - 3/n^2}{6 + 6/n^4 - 6/n^5} = \frac{6}{6} = 1$$

Also $\sum_{n=1}^{\infty} \frac{2n^2}{3n^5} = \frac{2}{3}\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges (p-series $p = 3 > 1$)

so the original series also $\boxed{\text{converges}}$