Math 152 - Fall 2013 ~~Quiz #9~~ **Score:** ________ out of 10.

Name: Key

Show all work clearly and in order. Please box your answers.

1. Use the **Ratio Test** to determine whether the series converges or diverges. If the test is inconclusive then say so.

$$\sum_{n=1}^{\infty} \frac{(n+1)!}{10^n}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+2)!}{10^{n+1}} \cdot \frac{10^n}{(n+1)!} = \lim_{n\to\infty} \frac{(n+2)\cdot\cancel{(n+1)!}\,\cancel{10^n}}{\cancel{10^n}\cdot 10^1\,\cancel{(n+1)!}} = \lim_{n\to\infty} \frac{n+2}{10}$$

$$= \infty > 1$$

Series $\boxed{\text{diverges}}$

2. Use the **Root Test** to determine whether the series converges or diverges. If the test is inconclusive then say so.

$$\sum_{n=1}^{\infty} \left(1+\frac{1}{n^2}\right)^n$$

$$\lim_{n\to\infty} (a_n)^{1/n} = \lim_{n\to\infty} \left[\left(1+\frac{1}{n^2}\right)^n\right]^{1/n} = \lim_{n\to\infty} \left(1+\frac{1}{n^2}\right) = 1+0 = 1$$

$\boxed{\text{No information}}$

(the test is inconclusive)

3. Use the **Alternating Series Test** to determine whether the series converges. If the test is inconclusive then say so.

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2+1}$$

(a) Show: $\left\{\frac{1}{n^2+1}\right\}$ is decreasing

$$f(x) = \frac{1}{x^2+1} \longrightarrow f'(x) = \frac{(x^2+1)(0) - 1(2x)}{(x^2+1)^2} = \frac{-2x}{(x^2+1)^2} < 0$$

$\ominus$ $\oplus$

✓

(b) $\lim_{n\to\infty} \frac{1}{n^2+1} = 0$ ✓

By AST., the series $\boxed{\text{converges}}$