TEST 1

Math 152 - Calculus II **Score:** ________ out of 100

2/8/2013 **Name:** ______________________________

Read all of the following information before starting the exam:

- You have 50 minutes to complete the exam.
- Show all work, clearly and in order, if you want to get full credit. Please make sure you read the directions for each problem. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Please $\boxed{\text{box/circle}}$ or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 8 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. Evaluate:

(a) $\int \sec^2(x)\sqrt{1+\tan(x)}dx$.

$$u = 1+\tan(x) \Rightarrow \frac{du}{dx} = \sec^2(x) \Longrightarrow dx = \frac{du}{\sec^2(x)}$$

$$\int \cancel{\sec^2(x)}\sqrt{u}\,\frac{du}{\cancel{\sec^2(x)}} = \int \sqrt{u}\,du = \int u^{1/2}\,du = \frac{u^{3/2}}{3/2} + C$$

$$= \boxed{\frac{2}{3}(1+\tan(x))^{3/2} + C}$$

(b) $\int \frac{\sin(2x)}{1+\cos(2x)}dx$.

$$u = 1+\cos(2x) \Longrightarrow \frac{du}{dx} = -\sin(2x)\cdot 2 \Longrightarrow dx = \frac{du}{-2\sin(2x)}$$

$$\int \frac{\cancel{\sin(2x)}}{u}\cdot\frac{du}{-2\cancel{\sin(2x)}} = -\frac{1}{2}\int \frac{1}{u}\,du = -\frac{1}{2}\ln|u| + C$$

$$= \boxed{-\frac{1}{2}\ln|1+\cos(2x)|}$$

or

$$\boxed{-\frac{1}{2}\ln(1+\cos(2x))}$$ since $1+\cos(2x) \geq 0$

2. Find the average value of $f(x) = \frac{e^x}{(1+e^x)^{1/3}}$ on $[1, 3]$.

$$f_{ave} = \frac{1}{3-1}\int_1^3 \frac{e^x}{(1+e^x)^{1/3}}\,dx$$

$$u = 1+e^x \Longrightarrow \frac{du}{dx} = e^x \Rightarrow dx = \frac{du}{e^x}$$

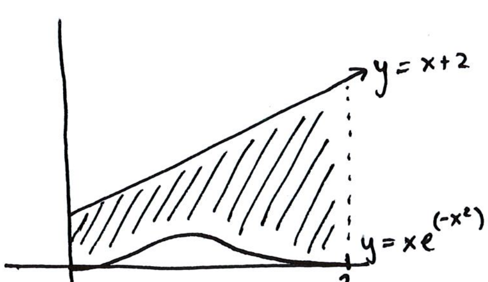
$u(1) = 1+e$

$u(3) = 1+e^3$

$$\frac{1}{2}\int_{1+e}^{1+e^3} \frac{\cancel{e^x}}{u^{1/3}}\cdot\frac{du}{\cancel{e^x}} = \frac{1}{2}\int_{1+e}^{1+e^3} u^{-1/3}\,du = \frac{1}{2}\left[\frac{u^{2/3}}{2/3}\right]_{1+e}^{1+e^3} = \boxed{\frac{3}{4}\left[(1+e^3)^{2/3} - (1+e)^{2/3}\right]}$$

≈ 3.92417

3. Find the area enclosed by the curves $y = x+2$, $y = xe^{(-x^2)}$, $x = 0$ and $x = 2$.



$$\text{Area} = \int_0^2 \left[(x+2) - xe^{(-x^2)}\right]dx$$

$$= \left[\frac{x^2}{2} + 2x\right]_0^2 - \int_0^2 xe^{(-x^2)}\,dx$$

$$u = -x^2 \Rightarrow \frac{du}{dx} = -2x \Rightarrow dx = \frac{du}{-2x}$$

$u(0) = 0$

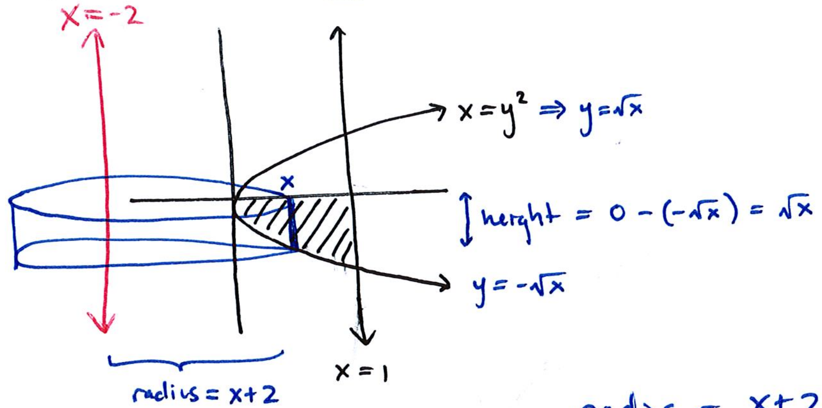
$u(2) = -4$

$$= \left[\left(\frac{4}{2}+4\right) - (0+0)\right] - \int_0^{-4} \cancel{x}e^u\cdot\frac{du}{-2\cancel{x}}$$

$$= 6 + \frac{1}{2}\int_0^{-4} e^u\,du = 6 + \frac{1}{2}\left[e^u\right]_0^{-4} = \boxed{\frac{1}{2}\left(11 + \frac{1}{e^4}\right)} \approx 5.509$$

4. Find the volume of the solid obtained by rotating the region bounded by $x = y^2$, $x = 1$ and $y \le 0$ about the line $x = -2$ using **any method**.

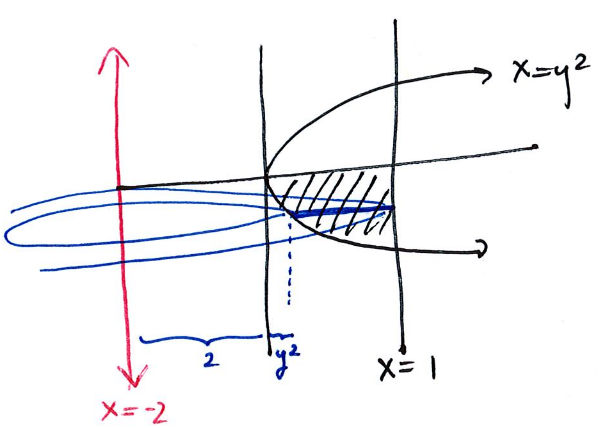
$\boxed{\text{SOLUTION 1}}$



$$\text{radius} = x+2$$
$$\text{height} = \sqrt{x}$$

$$V = \int_0^1 2\pi(\text{radius})(\text{height})\,dx$$
$$= \int_0^1 2\pi(x+2)(\sqrt{x})\,dx = 2\pi\int_0^1 \left(x^{3/2} + 2x^{1/2}\right)dx = 2\pi\left[\frac{x^{5/2}}{5/2} + \frac{2x^{3/2}}{3/2}\right]_0^1$$
$$= \boxed{\frac{52\pi}{15}}$$
$$\approx \boxed{10.8909}$$

$\boxed{\text{SOLUTION 2}}$



r = inner radius $= 2+y^2$

R = outer radius $= 3$

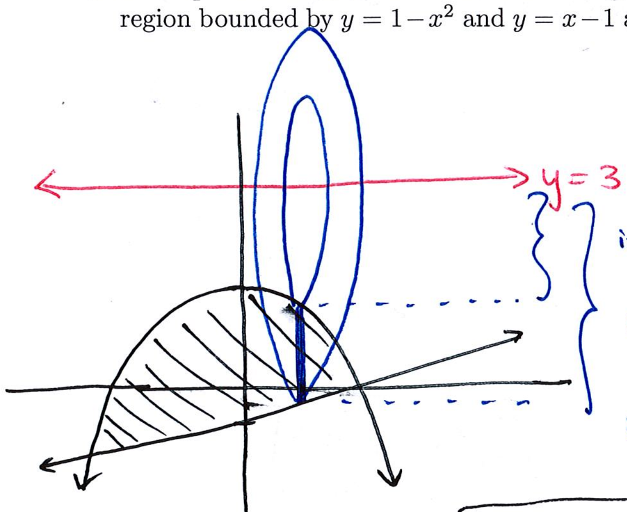
$r = 2+y^2$

$R = 2+1 = 3$

$$V = \int_{-1}^{0} \left[\pi(3)^2 - \pi(2+y^2)^2\right]dy$$
$$= \pi\int_{-1}^{0} 9 - (4+2y^2+y^4)\,dy$$
$$= \pi\int_{-1}^{0} (9-4-2y^2-y^4)\,dy$$
$$= \pi\int_{-1}^{0} (5-2y^2-y^4)\,dy$$
$$= \pi\left[5y - \frac{2y^3}{3} - \frac{y^5}{5}\right]_{-1}^{0}$$
$$= \boxed{\frac{52\pi}{15}} \approx \boxed{10.8909}$$

4. Find the volume of the solid obtained by rotating the region bounded by $x = y^2$, $x = 1$ and $y \leq 0$ about the line $x = -2$ using **any method**.

5. **Set up but do not evaluate the integral** for the volume of the solid obtained by rotating the region bounded by $y = 1 - x^2$ and $y = x - 1$ about the line $y = 3$ **using the Washer/Disk Method**.



inner radius $= r = 3 - (1 - x^2) = 3 - 1 + x^2 = 2 + x^2$

outer radius $= R = 3 - (x-1) = 4 - x$

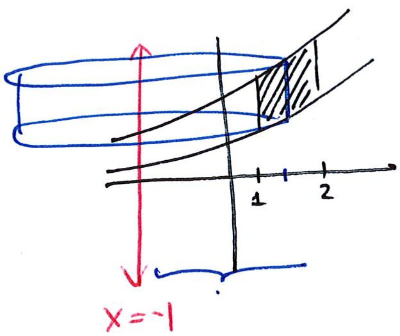
int pts: $1 - x^2 = x - 1 \Rightarrow x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2 \mid x = 1$

$$\boxed{V = \int_{-2}^{1} \pi\left[3-(x-1)\right]^2 - \pi\left[3-(1-x^2)\right]^2 \, dx}$$

6. **Set up but do not evaluate the integral** for the volume of the solid obtained by rotating the region bounded by $y = e^{2x} + 1$, $y = e^x$, $x = 1$ and $x = 2$ about the line $x = -1$ **using the (cylindrical) Shell Method.**



radius $= x+1$

height $= (e^{2x}+1-e^x)$

$$\boxed{V = \int_1^2 2\pi\,(x+1)\left(e^{2x}+1-e^x\right)dx}$$

7. **Set up but do not evaluate the integral** for the length of the curve $y = \sin^2(x)$ from $x = 1$ to $x = 3$.

$y = (\sin(x))^2$

$y' = 2\sin(x)\cos(x)$

$$\boxed{L = \int_1^3 \sqrt{1 + [2\sin(x)\cos(x)]^2}\,dx}$$

8. **Set up but do not evaluate the integral** for the surface area of the solid formed by rotating the portion of curve $y = e^{(1+3x^2)}$ from $x = 1$ to $x = 3$ about the x-axis.

$y' = (6x)e^{(1+3x^2)}$

$$\boxed{S = \int_1^3 2\pi\left(e^{(1+3x^2)}\right)\sqrt{1 + \left[6x\,e^{(1+3x^2)}\right]^2}\,dx}$$