TEST 2

Math 152 - Calculus II

Score: ________ out of 100

3/1/2013

Name: Key

Read all of the following information before starting the exam:

- You have 50 minutes to complete the exam.
- Show all work, clearly and in order, if you want to get full credit. Please make sure you read the directions for each problem. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Please $\boxed{\text{box/circle}}$ or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 8 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. Evaluate $\int x^2 e^{-x} dx$.

$u = x^2 \qquad dv = e^{-x}$

$du = 2x \qquad v = -e^{-x}$

$x^2(-e^{-x}) - \int (-e^{-x}) 2x \, dx$

$-x^2 e^{-x} + 2\int x e^{-x} dx$

$u = x \qquad dv = e^{-x}$

$du = 1 \qquad v = -e^{-x}$

$-x^2 e^{-x} + 2\left[x(-e^{-x}) - \int(-e^{-x})dx\right] = -x^2e^{-x} + 2x(-e^{-x}) + 2(-e^{-x}) + C$

$= \boxed{-x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C}$

2. Evaluate $\int x \sec(x) \tan(x) dx$.

$u = x \qquad dv = \sec(x)\tan(x)$

$du = 1 \qquad v = \sec(x)$

$x\sec(x) - \int \sec(x) \, dx$

$\boxed{x\sec(x) - \ln|\sec(x) + \tan(x)| + C}$

3. Evaluate $\int \cos^5(7x) \sin^4(7x) dx$.

$\int \cos(7x) \cos^4(7x) \sin^4(7x) dx$

$\int \cos(7x)(1-\sin^2(7x))^2 \sin^4(7x) dx$

$u = \sin(7x) \Rightarrow \frac{du}{dx} = 7\cos(7x) \Rightarrow dx = \frac{du}{7\cos(7x)}$

$\frac{1}{7}\int (1-u^2)^2 u^4 \, du = \frac{1}{7}\int(1-2u^2+u^4)u^4 = \frac{1}{7}\int(u^4 - 2u^6 + u^8) du$

$= \frac{1}{7}\left[\frac{u^5}{5} - \frac{2u^7}{7} + \frac{u^9}{9}\right] + C$

$= \boxed{\frac{1}{7}\left[\frac{\sin^5(7x)}{5} - \frac{2}{7}\sin^7(7x) + \frac{\sin^9(7x)}{9}\right] + C}$

4. Evaluate $\int \frac{x^2}{\sqrt{16-x^2}}dx.$

$x = 4\sin\theta \qquad (\theta \in [-\frac{\pi}{2},\frac{\pi}{2}])$

$\frac{dx}{d\theta} = +4\cos\theta \implies dx = +4\cos\theta\, d\theta$

$$\int \frac{(4\sin\theta)^2}{\sqrt{16-(4\sin\theta)^2}} + 4\cos\theta\, d\theta = \int \frac{16\sin^2\theta}{\cancel{4\cos\theta}} \cancel{4\cos\theta}\, d\theta$$

$$= 16\int\left(\frac{1}{2}-\frac{1}{2}\cos(2\theta)\right)d\theta$$

$$= 8\int(1-\cos(2\theta))\,d\theta$$

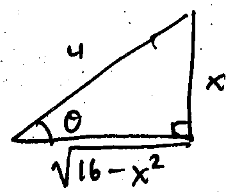
$$= 8\left(\theta - \frac{\sin(2\theta)}{2}\right) + C$$

$$= 8\left(\theta - \frac{\cancel{2}\sin\theta\cos\theta}{\cancel{2}}\right) + C$$

$$= 8\left(\sin^{-1}\left(\frac{x}{4}\right) - \left(\frac{x}{4}\right)\left(\frac{\sqrt{16-x^2}}{4}\right)\right) + C$$

$$= \boxed{8\sin^{-1}\left(\frac{x}{4}\right) - \frac{1}{2}x\sqrt{16-x^2} + C}$$

$\sin\theta = \frac{x}{4}$



$\cos\theta = \frac{\sqrt{16-x^2}}{4}$

5. Evaluate $\int \frac{2x-41}{x^2+x-12}dx.$

$$\int \frac{2x-41}{(x+4)(x-3)}dx$$

$$\frac{2x-41}{(x+4)(x-3)} = \frac{A}{x+4} + \frac{B}{x-3}$$

$$= \frac{A(x-3)+B(x+4)}{(x+4)(x-3)}$$

$$2x-41 = Ax - 3A + Bx + 4B$$

$A+B = 2$ AND $-41 = -3A + 4B$

$A = 2-B \longrightarrow -41 = -3(2-B)+4B$

$-41 = -6+3B+4B$

$-35 = 7B$

$A = 7 \longleftarrow B = -5$

$$\int \frac{2x-41}{x^2+x-12}dx = \int\left(\frac{7}{x+4} + \frac{-5}{x-3}\right)dx = \boxed{7\ln|x+4| - 5\ln|x-3| + C}$$

6. Evaluate $\int \sec^4(3x)\tan^2(3x)dx$.

$$\int \sec^2(3x)\,\sec^2(3x)\,\tan^2(3x)\,dx$$

$$\int \sec^2(3x)\left(\tan^2(3x)+1\right)\tan^2(3x)\,dx$$

$$u = \tan(3x) \implies \frac{du}{dx} = \sec^2(3x)\cdot 3$$

$$\frac{1}{3}\int (u^2+1)u^2\,du$$

$$\frac{1}{3}\int (u^4+u^2)\,du$$

$$\frac{1}{3}\left[\frac{u^5}{5}+\frac{u^3}{3}\right]+C = \boxed{\frac{1}{3}\left[\frac{\tan^5(3x)}{5}+\frac{\tan^3(3x)}{3}\right]+C}$$

7. Use polynomial long division to evaluate $\int \frac{x^4-5}{x+1}dx$.

$$\begin{array}{r|l} & x^3 - x^2 + x - 1 \\ x+1 & x^4 \qquad\qquad -5 \\ & -(x^4+x^3) \\ & -x^3 \\ & -(-x^3-x^2) \\ & x^2 \\ & -(x^2+x) \\ & -x-5 \\ & -(-x-1) \\ & -4 \end{array}$$

$$\int \frac{x^4-5}{x+1}dx = \int\left(x^3 - x^2 + x - 1 - \frac{4}{x+1}\right)dx$$

$$= \boxed{\frac{x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2} - x - 4\ln|x+1| + C}$$

8. Write out the FORM of the partial fraction decomposition for the following (DO NOT find the numerical values for the unknown coefficients).

(a) $\frac{4x^3-1}{x^2(x-4)^2(x+3)} = \boxed{\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-4}+\frac{D}{(x-4)^2}+\frac{E}{(x+3)}}$

(b) $\frac{x+10}{x^3+5x^2+6x} = \frac{x+10}{x(x^2+5x+6)} = \frac{x+10}{x(x+3)(x+2)} = \boxed{\frac{A}{x}+\frac{B}{x+3}+\frac{C}{x+2}}$

(c) $\frac{2x^3+4x-15}{x(x-1)(x^2-1)^2} = \frac{2x^3+4x-15}{x(x-1)^3(x+1)^2} = \boxed{\frac{A}{x}+\frac{B}{x-1}+\frac{C}{(x-1)^2}+\frac{D}{(x-1)^3}+\frac{E}{x+1}+\frac{F}{(x+1)^2}}$