TEST 4 PRACTICE PROBLEMS
CALCULUS II (MATH 152)
SPRING 2013

(1) Use the **Ratio Test** to determine whether the series converges or diverges. If the test is inconclusive then say so.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{7^k}{k!}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n 4^{2n}}{(2n)!}$

(c) $\displaystyle\sum_{k=1}^{\infty} \frac{k^2}{(2k+1)!}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{7^n}{n 2^{2n}}$

(2) Use the **Root Test** to determine whether the series converges or diverges. If the test is inconclusive then say so.

(a) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{7k^2-5}{5k^2+k-1}\right)^k$

(b) $\displaystyle\sum_{k=1}^{\infty} \left(1+\frac{1}{k}\right)^k$

(c) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{\tan^{-1}(k)}{\pi}\right)^k$

(d) $\displaystyle\sum_{k=1}^{\infty} \frac{n^{2n}}{3^{3n}}$

(3) Use the **Alternating Series Test** to determine whether the series converges. If the test is inconclusive then say so.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2+e^k}$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2+1}$

(c) $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^{k+1}\ln(k)}{k}$

(d) $\displaystyle\sum_{k=2}^{\infty} \frac{\cos(n\pi)}{k\sqrt{k}}$

(4) Determine whether the following series converges or diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)^n}{n^{2n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{4^n+1}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)\sin(2n)\sin(3n)\cdots\sin(2013n)}{n^{2013}}$

(5) Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\frac{n}{n^2+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{2/3}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)\cos(3n)}{n^8}$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln n}$

(e) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\frac{n^3}{2n^3+1}$

(f) $\displaystyle\sum_{n=1}^{\infty} \sin(n\pi/2)$

(6) Using the formula, set up a table and find the first FOUR nonzero terms of the Maclaurin series for

(a) $f(x) = \dfrac{5}{1+2x} = 5(1+2x)^{-1}$
(b) $f(x) = e^{-2x}$
(c) $f(x) = \cos(2x-1)$
(d) $f(x) = \tan^{-1}(x)$

(7) Using the formula, set up a table and find the first THREE nonzero terms of the Taylor series about the given x_0 for

(a) $f(x) = \tan^{-1}(x)$, $x_0 = 1$.
(b) $f(x) = e^{-x}$, $x_0 = 5$.
(c) $f(x) = \cos(x)$, $x_0 = \pi/2$.
(d) $f(x) = \sin\left(\dfrac{\pi}{2}x\right)$, $x_0 = 1$.

(8) Find the radius and interval of convergence for the power series

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+3)^n}{n^2}$
(b) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n+1}$
(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{\sqrt{n}}$
(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$
(e) $\displaystyle\sum_{n=1}^{\infty} (n+2)!(x-5)^n$
(f) $\displaystyle\sum_{n=2}^{\infty} \frac{x^n}{\ln n}$
(g) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2 x^n}{2^n}$
(h) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{3^n \ln n}$

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