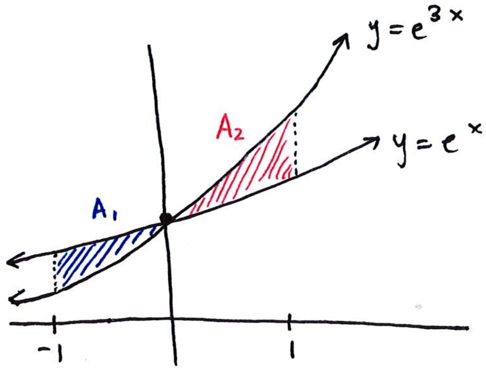
Math 152 - Spring 2013 - Quiz #1 Score: ________ out of 10.

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Find the area of the region bounded by $y = e^x$ and $y = e^{3x}$, $x = -1$ and $x = 1$.



$$A = A_1 + A_2$$

$$= \int_{-1}^{0} (e^x - e^{3x})\,dx + \int_0^1 (e^{3x} - e^x)\,dx$$

u-sub $u = 3x$ u-sub $u = 3x$

$$= \left[e^x - \frac{e^{3x}}{3} \right]_{-1}^{0} + \left[\frac{e^{3x}}{3} - e^x \right]_0^1$$

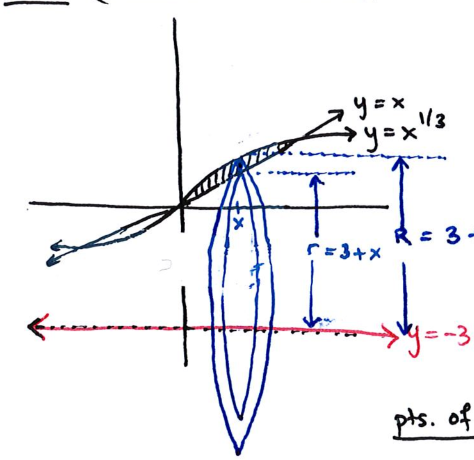
$$= \left[\left(e^0 - \frac{e^0}{3}\right) - \left(e^{-1} - \frac{e^{-3}}{3}\right)\right] + \left[\left(\frac{e^3}{3} - e^1\right) - \left(\frac{e^0}{3} - e^0\right)\right]$$

$$= \boxed{\frac{4}{3} - \frac{1}{e} + \frac{1}{3e^3} + \frac{e^3}{3} - e}$$

$$\approx 4.959$$

2. **Set up but do not evaluate the integral** for the volume of the solid obtained by rotating the region bounded by $y = x^{1/3}$, $y = x$ and $x \geq 0$ about the line $y = -3$.

SOL 1: (Washer / Disk Method)



pts. of intersection:

$x^{1/3} = x$

$x = x^3$

$x^3 - x = 0$

$x(x^2 - 1) = 0$

$x(x-1)(x+1) = 0$

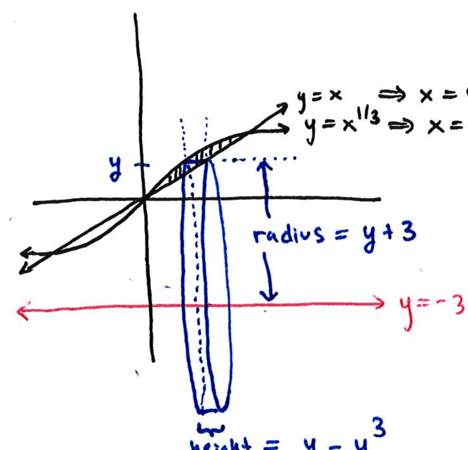
$\underbrace{x = 0 \mid x = 1}_{x \geq 0 \text{ is required (given)}} \mid x = -1$

r = inner radius = $3 + x$

R = outer radius = $3 + x^{1/3}$

$$\boxed{V = \int_0^1 \left[\pi (3 + x^{1/3})^2 - \pi (3 + x)^2\right] dx}$$

SOL 2: (Shell Method)



pts. of intersection

$y^3 = y$

$y^3 - y = 0$

$y(y^2 - 1) = 0$

$y(y-1)(y+1) = 0$

$\underbrace{y = 0 \mid y = 1}_{y \geq 0} \mid y = -1$

height = $y - y^3$

radius = $y + 3$

$$\boxed{V = \int_0^1 2\pi (y+3)(y - y^3)\,dy}$$