

Name: _____

key

SOLVE ONE OF THE FOLLOWING:

Please indicate which problem you do NOT want me to grade by putting a GIANT X through it, otherwise I will grade the first side worked on:

Show all work clearly and in order. Please box your answers. 10 minutes.

1. It takes 3 Joules of work to stretch a spring from its natural length to 14 meters beyond its natural length. What is the force (in Newtons) that holds the spring stretched at the same distance 14 meters?

Since we have a spring we know $F(x) = Kx$.

The first sentence tell us that

$$W = \int_a^b F(x) dx$$

$$3 = \int_0^{14} Kx dx = \left[\frac{Kx^2}{2} \right]_0^{14} = \frac{K}{2} [(14)^2 - 0^2]$$

$$3 = \frac{(14)^2 K}{2}$$

$$\text{Thus, } k = \frac{6}{14^2}$$

This means the force function is $F(x) = \left(\frac{6}{14^2}\right)x$

$$\text{Now, } F(14) = \frac{6}{(14)^2} \cdot 14 = \frac{6}{14}$$

$$= \boxed{\frac{3}{7} \text{ N}}$$

2. Evaluate $\int x \sec^2(x) dx$.

↑ algebraic ↑ trig

Following LI(A)TE

↑ algebraic is first, so

$$\begin{array}{l} u = x \\ du = 1 dx \end{array} \quad \left\| \begin{array}{l} dv = \sec^2(x) \\ v = \tan(x) \end{array} \right.$$

$$\int x \sec^2(x) dx = x \tan(x) - \int \tan(x) dx$$

$$= x \tan(x) - \int \frac{\sin(x)}{\cos(x)} dx$$

$$= x \tan(x) - \int \frac{\sin(x)}{u} \cdot \frac{du}{(-\sin(x))}$$

$$= x \tan(x) + \int \frac{1}{u} du$$

$$= x \tan(x) + \ln|u| + C = \boxed{x \tan(x) + \ln|\cos(x)| + C}$$

$$u = \cos(x) \Rightarrow \frac{du}{dx} = -\sin(x) \\ dx = \frac{du}{-\sin(x)}$$