Math 152 - Spring 2013 - Quiz #5 **Score:** ________ out of 10.

Name: key

Show all work clearly and in order. Please box your answers.

1. Determine whether the sequence converges, and if so find its limit.

(a) $\left\{\dfrac{(-1)^{n+1}}{5n^3}\right\}_{n=1}^{\infty}$

$\boxed{\text{SOL 1}}$ use the squeeze thm

Notice that

$$\frac{-1}{5n^3} \le \frac{(-1)^{n+1}}{5n^3} \le \frac{1}{5n^3}.$$

Also, $\lim\limits_{n\to\infty} \dfrac{-1}{5n^3} = 0$ AND

$\lim\limits_{n\to\infty} \dfrac{1}{5n^3} = 0$.

Hence, $\lim\limits_{n\to\infty} \dfrac{(-1)^{n+1}}{5n^3} = \boxed{0}$, by the squeeze Thm. $\boxed{\text{converges}}$

$\boxed{\text{SOL 2}}$ Recall, Thm. If $\lim\limits_{n\to\infty} |a_n| = 0$ then $\lim\limits_{n\to\infty} a_n = 0$

Notice that $\lim\limits_{n\to\infty} |a_n| = \lim\limits_{n\to\infty} \left|\dfrac{(-1)^{n+1}}{5n^3}\right|$

$= \lim\limits_{n\to\infty} \dfrac{1}{5n^3}$

$= 0$

Hence, by the Thm. above

$\lim\limits_{n\to\infty} \dfrac{(-1)^{n+1}}{5n^3} = \boxed{0}$ $\boxed{\text{converges}}$

(b) $\left\{\dfrac{\ln(\ln(n))}{n}\right\}_{n=2}^{\infty}$

$\boxed{\text{SOL}}$ $\lim\limits_{n\to\infty} \dfrac{\ln(\ln(n))}{n}$ ← goes to ∞ ← goes to ∞ so let's try L'Hôpital's Rule

$$\lim_{x\to\infty} \frac{\ln(\ln(x))}{x} \overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{d}{dx}\left(\ln(\ln(x))\right)}{\frac{d}{dx}(x)} = \lim_{x\to\infty} \frac{\frac{1}{\ln(x)}\cdot\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{\ln(x)\cdot x} = 0.$$

Therefore,

$\lim\limits_{n\to\infty} \dfrac{\ln(\ln(n))}{n} = \boxed{0}$ $\boxed{\text{converges.}}$

2. Show that the given sequence is strictly increasing or strictly decreasing.

3 ways!

$$\left\{\frac{6n}{7n+2}\right\}_{n=1}^{\infty}$$

$\boxed{\text{SOL 1}}$ $f(x) = \dfrac{6x}{7x+2}$ (where $x \ge 1$)

$f'(x) = \dfrac{(7x+2)6 - 6x(7)}{(7x+2)^2}$

$= \dfrac{6\cdot 7x + 12 - 6\cdot 7x}{(7x+2)^2}$

$= \dfrac{12}{(7x+2)^2} > 0$ $\oplus$ $\oplus$

Since $f'(x) > 0 \Rightarrow$ $f(x)$ is strictly increasing

SO →

$\boxed{\text{SOL 2}}$ $a_{n+1} - a_n$

$= \dfrac{6(n+1)}{7(n+1)+2} - \dfrac{6n}{7n+2}$

$= \dfrac{6n+6}{7n+9} - \dfrac{6n}{7n+2}$

$= \dfrac{(6n+6)(7n+2) - (6n)(7n+9)}{(7n+9)(7n+2)}$

$= \dfrac{12}{(7n+9)(7n+2)} > 0$ $\oplus$ $\oplus$ $\oplus$

so ↓

$\boxed{\text{SOL 3}}$ $\dfrac{a_{n+1}}{a_n} = \dfrac{\frac{6(n+1)}{7(n+1)+2}}{\frac{6n}{7n+2}}$

$= \dfrac{(6n+6)}{(7n+9)}\cdot\dfrac{(7n+2)}{(6n)}$

$= \dfrac{42n^2 + 54n + 12}{42n^2 + 54n}$

> 1

numerator is 12 more than the denom

Hence,

so ←

The sequence is strictly $\boxed{\text{increasing}}$