Math 152 - Spring 2013 - Quiz #6 **Score:** ________ out of 10.

Name: key

Show all work clearly and in order. Please box your answers.

1. Each series below is geometric. Determine both a and r. Then decide whether the series converges or diverges. If the series converges, then find its sum. If it diverges, write "NO SUM."

(a) $\sum_{k=1}^{\infty}\left(-\frac{9}{5}\right)^{k-1}$

$a = \boxed{1}$

$r = \boxed{-9/5}$ $|r| = \frac{9}{5} \geq 1$ series diverges

sum= $\boxed{\text{NO SUM}}$

(b) $\sum_{k=0}^{\infty}(-1)^{k+1}\frac{2^{3k}}{9^{k+1}} = \sum_{k=0}^{\infty}(-1)(-1)^k \frac{8^k}{9 \cdot 9^k} = \sum_{k=0}^{\infty}\left(-\frac{1}{9}\right)\left(\frac{-8}{9}\right)^k$

$a = \boxed{-1/9}$

$r = \boxed{-8/9}$ $|r| = 8/9 < 1$ converges to

sum= $\boxed{-1/17}$ $\frac{-1/9}{1-(-8/9)} = \frac{-1/9}{17/9} = \frac{-1}{17}$

2. Use the Divergence Test to determine whether the given series diverges. If the test yields no conclusion, then be sure to say so. You must set up, evaluate, and interpret the correct limit to earn credit.

$$\sum_{n=1}^{\infty}\frac{4n^5}{2n^5+3n-1}$$

$$\lim_{n\to\infty}\frac{4n^5}{2n^5+3n-1} = \lim_{n\to\infty}\frac{(4n^5)\left(\frac{1}{n^5}\right)}{(2n^5+3n-1)\left(\frac{1}{n^5}\right)} = \lim_{n\to\infty}\frac{4}{2+\frac{3}{n^4}-\frac{1}{n^5}}$$

$$= \frac{4}{2+0-0} = 2 \neq 0$$

test for divergence says this series $\boxed{\text{diverges}}$

3. Use the partial fraction expansion

$$\frac{1}{k^2+3k+2} = \frac{1}{k+1} - \frac{1}{k+2}$$

to determine whether the series

$$\sum_{k=1}^{\infty}\frac{1}{k^2+3k+2}$$

converges of diverges. If it converges, find the sum.

$$S_n = \sum_{k=1}^{n}\frac{1}{k^2+3k+2} = \sum_{k=1}^{n}\left(\frac{1}{k+1}-\frac{1}{k+2}\right) = \left(\frac{1}{2}-\cancel{\frac{1}{3}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{4}}\right)+\cdots+\left(\cancel{\frac{1}{n}}-\cancel{\frac{1}{n+1}}\right)+\left(\cancel{\frac{1}{n+1}}-\frac{1}{n+2}\right)$$

$$= \frac{1}{2} - \frac{1}{n+2}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\frac{1}{2}-\frac{1}{n+2}\right) = \frac{1}{2}+0 = \frac{1}{2}$$

$\therefore \sum_{k=1}^{\infty}\frac{1}{k^2+3k+2}$ $\boxed{\text{converges}}$ and its sum is $\boxed{1/2}$