Math 152 - Spring 2013 - Quiz #7 **Score:** ________ out of 10.

Name: key

Show all work clearly and in order. Please box your answers.

1. Use the **Ratio Test** to determine whether the series converges or diverges. If the test is inconclusive then say so.

$$\sum_{k=1}^{\infty} \frac{7^k}{k!}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{7^{n+1}}{(n+1)!}}{\frac{7^n}{n!}} = \lim_{n\to\infty} \frac{\cancel{7^n}\,7}{(n+1)\cancel{n!}} \cdot \frac{\cancel{n!}}{\cancel{7^n}} = \lim_{n\to\infty} \frac{7}{n+1}$$

$$= 0 < 1$$

series $\boxed{\text{converges}}$

2. Use the **Root Test** to determine whether the series converges or diverges. If the test is inconclusive then say so.

$$\sum_{k=1}^{\infty} \left(\frac{7k^2-5}{5k^2+k-1}\right)^k$$

$$\lim_{n\to\infty} (a_n)^{1/n} = \lim_{n\to\infty} \left[\left(\frac{7n^2-5}{5n^2+n-1}\right)^n\right]^{1/n} = \lim_{n\to\infty} \frac{7n^2-5}{5n^2+n-1} = \frac{7}{5} > 1$$

series $\boxed{\text{diverges}}$

3. Use the **Alternating Series Test** to determine whether the series converges. If the test is inconclusive then say so.

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2+e^k}$$

(a) Show $\left\{\frac{1}{2+e^k}\right\}$ is decreasing:

$$f(x) = \frac{1}{2+e^x} \Rightarrow f'(x) = \frac{-e^x}{(2+e^x)^2} < 0$$ so decreasing ✓

(b) $$\lim_{n\to\infty} \frac{1}{2+e^n} = 0$$ ✓

Hence, by the Alternating Series test the series $\boxed{\text{converges}}$