Math 152 - Spring 2013 - Quiz #8 **Score:** ________ out of 10.

Name: Key

PICK ONE OF THE FOLLOWING:

Please indicate which 2 problems you do NOT want me to grade by putting an X through each of them, otherwise I will grade the first problem worked on:

Show all work clearly and in order. Please box your answers.

1. Using the formula, set up a table and find the first FOUR nonzero terms of the Maclaurin series for

$$f(x) = \frac{1}{1-3x} = (1-3x)^{-1}.$$

n	$f^{(n)}(x)$	$f^{(n)}(0)$	$f^{(n)}(0)/n!$
0	$(1-3x)^{-1}$	1	$1/0! = 1$
1	$(-1)(1-3x)^{-2}(-3)$	$(-1)(-3) = 3$	$3/1! = 3$
2	$(-1)(-2)(1-3x)^{-3}(-3)(-3)$	$1\cdot 2\cdot 3^2$	$\frac{1\cdot 2\cdot 3^2}{2!} = 3^2$
3	$(-1)(-2)(-3)(1-3x)^{-4}(-3)(-3)(-3)$	$1\cdot 2\cdot 3\cdot 3^3$	$\frac{1\cdot 2\cdot 3\cdot 3^3}{3!} = 3^3$

$$\boxed{1 + 3x + 9x^2 + 27x^3 + \cdots}$$

or $\sum_{k=0}^{\infty} 3^k x^k$

2. Using the formula, set up a table and find the first THREE nonzero terms of the Taylor series about $x_0 = 1$ for

$$f(x) = \tan^{-1}(x).$$

n	$f^{(n)}(x)$	$f^{(n)}(1)$	$\frac{f^{(n)}(1)}{n!}$
0	$\tan^{-1}(x)$	$\tan^{-1}(1) = \frac{\pi}{4}$	$\frac{\pi/4}{0!} = \frac{\pi}{4}$
1	$\frac{1}{1+x^2}$	$\frac{1}{2}$	$\frac{1/2}{1!} = \frac{1}{2}$
2	$\frac{-2x}{(1+x^2)^2}$	$\frac{-2}{4} = -\frac{1}{2}$	$\frac{-1/2}{2!} = \frac{-1}{4}$

$$\boxed{\frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2 + \cdots}$$

3. Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{n^2+1}$$

SOL:

Try Ratio Test for Absolute convergence:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^n(n+1)}{(n+1)^2+1}\cdot\frac{(n^2+1)}{n(-1)^{n-1}}\right|$$

$$= \lim_{n\to\infty}\frac{(n+1)(n^2+1)}{((n^2+2n+1)+1)n}$$

$$= \lim_{n\to\infty}\frac{n^3+n^2+n+1}{n^3+2n^2+2n}$$

$$= 1$$ NO INFO! (try another test for absolute convergence.)

Look at $\sum|a_n|$:

$$\sum_{n=1}^{\infty}\left|\frac{(-1)^{n-1}n}{n^2+1}\right| = \sum_{n=1}^{\infty}\frac{n}{n^2+1}$$

looks like limit comparison will work.

$$a_n = \frac{n}{n^2+1} \qquad b_n = \frac{n}{n^2} = \frac{1}{n}$$

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\left(\frac{n}{n^2+1}\right)\left(\frac{n}{1}\right) = \lim_{n\to\infty}\frac{n^2}{n^2+1} = 1$$

finite and positive!

Since $\sum_{n=1}^{\infty}b_n = \sum_{n=1}^{\infty}\frac{1}{n}$ diverges (harmonic series)

$\sum_{n=1}^{\infty}a_n = \sum_{n=1}^{\infty}\frac{n}{n^2+1}$ also diverges.

So the original series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{n^2+1}$ diverges absolutely

Now test the original series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{n^2+1}$. This is alternating, so let's try the alternating series test:

(a) Show $\left\{\frac{n}{n^2+1}\right\}$ is decreasing: $f(x)=\frac{x}{x^2+1} \Rightarrow f'(x)=\frac{x^2+1-2x^2}{(x^2+1)^2}$

$$= \frac{1-x^2}{(x^2+1)^2} < 0 \quad (\text{if } x>1)$$ ✓

(b) $\lim_{n\to\infty}\frac{n\left(\frac{1}{n^2}\right)}{n^2+1\left(\frac{1}{n^2}\right)} \lim_{n\to\infty}\frac{1/n}{1+1/n^2} = \frac{0}{1+0} = 0$ ✓

So original series $\boxed{\text{conditionally converges}}$