TEST 3 PRACTICE PROBLEMS
CALCULUS II (MATH 152)
SPRING 2013

(1) Write out the FORM of the partial fraction decomposition for the following (DO NOT find the numerical values for the unknown coefficients).

(a) $\dfrac{x^3+x^2+1}{x(x-1)(x^2+1)^2} =$

(b) $\dfrac{x^2+10}{x^3(x^2+4)} =$

(c) $\dfrac{4x-1}{(x^2+x+11)(x^2+9)} =$

(2) Evaluate

(a) $\displaystyle\int_{-\infty}^{0} e^{5x}\,dx.$

(b) $\displaystyle\int_{-2}^{\infty} \frac{1}{1+x^2}dx.$

(c) $\displaystyle\int_{4}^{5} \frac{1}{(x-5)^3}dx.$

(d) $\displaystyle\int_{0}^{4} \frac{1}{x-3}dx.$

(3) Determine whether the sequence converges, and if so find its limit.

(a) $\left\{\dfrac{2n^3+1}{3n^3+2n^2-4}\right\}_{n=1}^{\infty}.$

(b) $\left\{\dfrac{(\ln n)^2}{3n}\right\}_{n=1}^{\infty}.$

(c) $\left\{\dfrac{\cos(n)}{n^2}\right\}_{n=1}^{\infty}.$

(d) $\left\{\dfrac{(-1)^n}{\sqrt{n}}\right\}_{n=1}^{\infty}.$

(4) Determine whether the following sequences are strictly increasing or strictly decreasing.

(a) $\left\{\dfrac{5n}{3n+1}\right\}_{n=1}^{\infty}$

(b) $\left\{\dfrac{\ln(n)}{n}\right\}_{n=1}^{\infty}$

(c) $\left\{\dfrac{n}{e^{4n}}\right\}_{n=1}^{\infty}$

(d) $\left\{\tan^{-1}(2n)\right\}_{n=1}^{\infty}$

(5) Each series below is geometric. Determine both a and r. Then decide whether the series converges or diverges. If the series converges, then find its sum.

(a) $\displaystyle\sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^{k+2}$

(b) $\displaystyle\sum_{k=0}^{\infty}(-1)^k\frac{3^{2k}}{7^{k+2}}$

(6) Use the Divergence Test on each of the following to determine whether the given series diverges. If the test yields no conclusion, then be sure to say so. You must set up, evaluate, and interpret the correct limit to earn credit.

(a) $\displaystyle\sum_{n=1}^{\infty}\cos\left(\frac{2}{n}\right)$

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{n^5+3}{3n^6-n^3+1}$

(c) $\displaystyle\sum_{n=1}^{\infty}\frac{e^n}{n^2}$

(d) $\displaystyle\sum_{n=1}^{\infty}\frac{\tan^{-1}(n)}{n}$

(7) Use the Integral Test to determine whether the given series converges or diverges. Clearly identify the function $f(x)$ you are embedding the sequence of terms into. You may assume that $f(x)$ is positive, decreasing and continuous for $x \geq 1$, so you do not need to verify this. Just use the integral test and state your conclusion.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n+3}}$

(8) Use the Comparison Test to determine whether the given series converges or diverges. Clearly write down an inequality, determine the convergence or divergence of your comparison series $\displaystyle\sum_{n=1}^{\infty} b_n$, and then write your conclusion.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{5}{n^{1/4}-1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^3(n)}{3^{n-1}}$

(9) Use the Limit Comparison Test to determine whether the given series converges or diverges. Clearly write down what a_n and b_n are, compute the appropriate limit, determine the convergence or divergence of your comparison series $\displaystyle\sum_{n=1}^{\infty} b_n$, and then write your conclusion.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{5n}{n^4-1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^{3/2}+1}$

(10) Determine whether the following series converges or diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{4}{4^n+1}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{8n}{n^2-n+2}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{n^2+1}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{4^{n+1}}{5^{2n}}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(1/2)^n+n^6}$

(11) Determine whether the following series converges or diverges. If the series converges, then find its sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$

(c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{3^n} - \frac{1}{3^{n+1}}\right)$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n-3^n}{(-7)^n}$

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