Math 271 - Fall 2012 - Quiz #3 Score: _____ out of 10.

Name: _____

Show all work clearly and in order. Please box your answers. 10 minutes.

PICK ONE OF THE FOLLOWING:

Please indicate which one you do NOT want me to grade by putting an X through it, otherwise I will grade the first one worked on:

1. Find an implicit AND explicit solution of the following initial-value problem:

$$\frac{dy}{dx} = 3x\sqrt{1-y^2}, \qquad y(0) = \frac{\sqrt{2}}{2}.$$

This D.E. is separable:

$$\frac{dy}{\sqrt{1-y^2}} = 3x\,dx \implies \int \frac{dy}{\sqrt{1-y^2}} = \int 3x\,dx$$

$$\sin^{-1}(y) = \frac{3x^2}{2} + C$$

Substitute $x=0$, $y=\frac{\sqrt{2}}{2}$:

$$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{3\cdot 0^2}{2} + C$$

$$\frac{\pi}{4} = 0 + C$$

$$C = \frac{\pi}{4}$$

Implicit Solution: $\sin^{-1}(y) = \frac{3x^2}{2} + \frac{\pi}{4}$

Explicit Solution: $y = \sin\left(\frac{3x^2}{2} + \frac{\pi}{4}\right)$

2. (a) Find the general explicit solution of

$$y' = \frac{2x(y+1)}{x^2+1}.$$

This DE is separable:

$$\frac{dy}{y+1} = \frac{2x}{x^2+1}\,dx \implies \int \frac{dy}{y+1} = \int \frac{2x}{x^2+1}\,dx$$

Substitute: $u = x^2+1 \Rightarrow \frac{du}{dx} = 2x$, so $dx = \frac{du}{2x}$

$$\ln|y+1| = \int \frac{2x}{u}\cdot\frac{du}{2x}$$

$$\ln|y+1| = \ln|u| + C$$

$$\ln|y+1| = \ln|x^2+1| + C$$

$x^2+1 \geq 0$ so, $\ln|y+1| = \ln(x^2+1) + C$

Now $e^{\ln|y+1|} = e^{(\ln(x^2+1)+C)} = e^{\ln(x^2+1)}\cdot e^C = A\,e^{\ln(x^2+1)}$ (where $A = e^C$)

so $|y+1| = A(x^2+1) \implies y+1 = B(x^2+1)$

Explicit Solution: $y = B(x^2+1) - 1$