

# TEST 3

Math 271 - Differential Equations

Score: \_\_\_\_\_ out of 100

4/23/2014

Name: \_\_\_\_\_

**Read all of the following information before starting the exam:**

- You have 50 minutes to complete the exam.
- Show all work, clearly and in order, if you want to get full credit. Please make sure you read the directions for each problem. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Please box/circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 5 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Good luck!

1.  $x = 0$  is an ordinary point of the differential equation:

$$y'' - xy' + 2y = 0.$$

Find two linearly independent power series solutions about  $x = 0$ . You should write down the first three nonzero terms of each series solution (unless your solution is a finite number of terms).

$$y_1 = \boxed{\phantom{\text{answer}}}$$

$$y_2 = \boxed{\phantom{\text{answer}}}$$

2. Find the following Laplace transforms

(a)  $\mathcal{L}\{2 + t^5 + e^{-3t}\}$

(b)  $\mathcal{L}\{e^{6t} \cos(3t)\}$

(c)  $\mathcal{L}\{4t\mathcal{U}(t - 9)\}$

3. Find the following **inverse** Laplace transforms

(a)  $\mathcal{L}^{-1} \left\{ \frac{1}{s^5} + \frac{s}{s^2 + 100} \right\}$

(b)  $\mathcal{L}^{-1} \left\{ \frac{1}{(s-4)^2 + 1} \right\}$

(c)  $\mathcal{L}^{-1} \left\{ e^{-5s} \left( \frac{6}{s^2 + 36} \right) \right\}$

4. Write  $f(t)$  in terms of unit step functions (Heaviside functions) if

$$f(t) = \begin{cases} 1, & 0 \leq t < \pi, \\ \ln(t), & \pi \leq t. \end{cases}$$

$f(t) =$
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5. Use the Laplace transform to solve the following initial value problem:

$$y'' + 9y = 10e^t, \quad y(0) = 0, \quad y'(0) = 0$$

$y(t) =$
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