Math 271 - Quiz #3 **Score:** ______ out of 10.

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Solve the following differential equation:

$$\frac{1}{4}y'' + y' + y = x^2 - 2x$$

$$y'' + 4y' + 4y = 4x^2 - 8x$$

Find y_c: $m^2 + 4m + 4 = 0$

$(m+2)(m+2) = 0$

$m = -2 \ / \ m = -2$

$y_c = C_1 e^{-2x} + C_2 x e^{-2x}$

Find y_p: (Method of undetermined coef.)

Looking at $g(x) = 4x^2 - 8x \longrightarrow y_p = Ax^2 + Bx + C$

No adjustment is necessary by looking at y_c.

$y_p' = 2Ax + B$

$y_p'' = 2A$

plug into the ODE: $2A + 4(2Ax + B) + 4(Ax^2 + Bx + C) = 4x^2 - 8x$

$2A + 8Ax + 4B + 4Ax^2 + 4Bx + 4C = 4x^2 - 8x$

$2A + 4B + 4C = 0$ | $8A + 4B = -8$ | $4A = 4$

$2 + (-16) + 4C = 0$ | $8 + 4B = -8$ | $A = 1$

$C = 14/4 = 7/2$ | $4B = -16 \rightarrow B = -4$

General Solution: $\boxed{y = C_1 e^{-2x} + C_2 x e^{-2x} + x^2 - 4x + \frac{7}{2}}$

2. Using the method of undetermined coefficients write the FORM for the particular solution (y_p) using the given value for $g(x)$ and the general solution of the associated homogeneous equation (y_c). Do NOT solve for the unknown constants, just write the form.

(a) $g(x) = 16e^{-x}$ and $y_c = C_1e^{-x} + C_2xe^{-x}$. so

$y_p = Ae^{-x}$ Adjust For y_c:

Form of y_p: $\boxed{y_p = Ax^2e^{-x}}$

(b) $g(x) = 2014\sin(5x)$ and $y_c = C_1\sin(5x) + C_2\cos(5x)$. so

$y_p = A\sin(5x) + B\cos(5x)$ Adjust For y_c:

Form of y_p: $\boxed{y_p = Ax\sin(5x) + Bx\cos(5x)}$

(c) $g(x) = 4e^{-2x}\cos 3x$ and $y_c = C_1e^{-2x}\sin(3x) + C_2e^{-2x}\cos(3x)$. so

$y_p = Ae^{-2x}\cos(3x) + Be^{-2x}\sin(3x)$ Adjust for y_c:

Form of y_p: $\boxed{y_p = Axe^{-2x}\cos(3x) + Bxe^{-2x}\sin(3x)}$