Math 271 - Quiz #9 **Score:** ______ out of 10.

Name: key

PICK ONE OF THE FOLLOWING:

Please indicate which problem you do NOT want me to grade by putting an X through it, otherwise I will grade the first problem worked on:

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Solve the following differential equation by variation of parameters:

$$y'' - 4y = \frac{e^{2x}}{x}$$

Find y_c:

$$y'' - 4y = 0$$
$$m^2 - 4 = 0$$
$$(m-2)(m+2) = 0$$
$$m = 2, \quad m = -2$$
$$y_c = c_1 \underbrace{e^{2x}}_{y_1} + c_2 \underbrace{e^{-2x}}_{y_2}$$

Find y_c: by variation of parameters

Standard Form ✓ $f(x) = \frac{e^{2x}}{x}$

$$W = \begin{vmatrix} e^{2x} & e^{-2x} \\ 2e^{2x} & -2e^{-2x} \end{vmatrix} = -2e^{2x}e^{-2x} - e^{2x}2e^{-2x}$$
$$= -2e^{0} - 2e^{4x}$$
$$= -2 - 2 = -4$$

$$u_1 = \int \frac{-y_2 f(x)}{W}\,dx = \int \frac{-e^{-2x}e^{2x}}{x(-4)} = +\frac{1}{4}\int \frac{1}{x} = \frac{1}{4}\ln(x)$$ ← may assume $x>0$ here.

$$u_2 = \int \frac{y_1 f(x)}{W}\,dx = \int \frac{e^{2x}e^{2x}}{x(-4)} = -\frac{1}{4}\int \frac{e^{4x}}{x}\,dx = -\frac{1}{4}Ei(4x)$$ ← Had integral 1. OR use series solution

$$= -\frac{1}{4}\int \frac{1}{x}\sum_{n=0}^{\infty}\frac{(4x)^n}{n!}\,dx$$
$$= -\frac{1}{4}\int \sum_{n=0}^{\infty}\frac{4^n x^{n-1}}{n!}\,dx = -\frac{1}{4}\left[\left(\sum_{n=1}^{\infty}\frac{4^n}{n!}\frac{x^n}{n}\right) + \ln(x)\right]$$

$$y = c_1e^{2x} + c_2e^{-2x} + \frac{1}{4}\ln(x)e^{2x} - \frac{1}{4}\left[\left(\sum_{n=1}^{\infty}\frac{4^n}{n!}\frac{x^n}{n}\right) + \ln(x)\right]$$

General Solution: $\boxed{y = c_1e^{2x} + c_2e^{-2x} + \frac{1}{4}\ln(x)e^{2x} - \frac{1}{4}Ei(4x)}$

2. Solve the following Cauchy-Euler equation:

$$x^2y'' + 10xy' + 8y = x^2$$

SOL: Find y_c: $a=1,\ b=10,\ c=8$

$$am^2 + (b-a)m + c = 0$$
$$m^2 + 9m + 8 = 0$$
$$(m+8)(m+1) = 0$$
$$m=-8,\ m=-1$$
$$y_c = \underbrace{C_1 x^{-1}}_{y_1} + \underbrace{C_2 x^{-8}}_{y_2}$$

Find y_p: by variation of parameters

Standard Form: $y'' + \frac{10}{x}y' + \frac{8}{x^2}y = \underbrace{1}_{f(x)}$

$$W = \begin{vmatrix} x^{-1} & x^{-8} \\ -x^{-2} & -8x^{-9} \end{vmatrix} = -8x^{-1}x^{-9} - (-x^{-2})(x^{-8})$$
$$= -8x^{-10} + x^{-10} = -7x^{-10}$$

$$u_1 = \int \frac{-y_2 f(x)}{W}dx = \int \frac{-x^{-8}(1)}{-7x^{-10}}dx = +\frac{1}{7}\int x^{-8-(-10)}dx$$
$$= +\frac{1}{7}\int x^2 dx$$
$$= +\frac{1}{7}\frac{x^3}{3} = -\frac{x^3}{21}$$

$$u_2 = \int \frac{y_1 f(x)}{W}dx = \int \frac{x^{-1}(1)}{-7x^{-10}}dx = -\frac{1}{7}\int x^9 dx = \frac{-x^{10}}{70}$$

$$y_p = u_1y_1 + u_2y_2 = \left(\frac{+x^3}{21}\right)(x^{-1}) + \left(\frac{-x^{10}}{70}\right)(x^{-8})$$
$$= \frac{+x^2}{21} + \frac{-x^2}{70}$$
$$= \frac{x^2}{30}$$

General Solution: $\boxed{y = C_1x^{-1} + C_2x^{-8} + \frac{x^2}{30}}$