

1. Let $x \in \mathbb{R}$, with $x \neq 0$. Prove that for all positive integers n ,

$$1 + x + x^2 + \cdots + x^n = \frac{1 - x^{n+1}}{1 - x}.$$

2. Let n be a positive integer. Suppose $x, y \in \mathbb{Z}$. Define the relation $\equiv$ by the following,

$$x \equiv y \text{ if and only if } x \bmod n = y \bmod n.$$

- (a) Show that $\equiv$ is an equivalence relation.
- (b) Suppose $n = 2$.
 - i. The equivalence class $[0]$ contains which elements?
 - ii. The equivalence class $[1]$ contains which elements?
 - iii. Are there any other equivalence classes?
- (c) Suppose $n = 3$.
 - i. How many distinct equivalence classes are there?
 - ii. Write out the elements for each equivalence class.
- (d) Using your work from parts (b) and (c), how can you use $\equiv$ to partition the integers?