

1. (a) Prove that the ring $\mathbb{Q}[x]/\langle x^2 - 3 \rangle$ is isomorphic to the ring $\mathbb{Q}[\sqrt{3}] = \{a + b\sqrt{3} \mid a, b \in \mathbb{Q}\}$.
(b) Is $\langle x^2 - 3 \rangle$ a prime ideal of $\mathbb{Q}[x]$? (Hint: $\mathbb{Q}[\sqrt{3}]$ is a field)
(c) Is $\langle x^2 - 3 \rangle$ a maximal ideal of $\mathbb{Q}[x]$? (Hint: $\mathbb{Q}[\sqrt{3}]$ is a field)
2. (a) Compute $(1 + x)^2$ in $\mathbb{Z}_2[x]$.
(b) Compute $(1 + x)^3$ in $\mathbb{Z}_3[x]$.
(c) Compute $(1 + x)^5$ in $\mathbb{Z}_5[x]$.
(d) Looking at the above you might make the conjecture that if p is prime, then $(1 + x)^p = 1 + x^p$ in $\mathbb{Z}_p[x]$. Prove that this conjecture is true! That is, prove Freshman's dream can come true (in $\mathbb{Z}_p[x]$). Bonus: song? See if you can outdo this: <http://www.youtube.com/watch?v=BipvGD-LCjU>
3. Tell me who you will write your biography on and what your presentation will be on.