Math 481 - Graded Problems #2 Name: ______________________________
Fall 2013 - Nathan Reff

1. Let n be a positive integer. Let
$$n\mathbb{Z} := \{nm \mid m \in \mathbb{Z}\}.$$
For example,
$$2\mathbb{Z} = \{2m \mid m \in \mathbb{Z}\} = \{\ldots, -4, -2, 0, 2, 4, \ldots\}.$$
   (a) Show that addition is a binary operation on $n\mathbb{Z}$.
   (b) Show that $(n\mathbb{Z}, +)$ is a group.
   (c) Is multiplication a binary operation on $n\mathbb{Z}$? Prove your answer is correct!
   (d) Is $(n\mathbb{Z}, \times)$ a group? Prove your answer is correct!

2. ♠ (Cool problem for fun).

   Suppose A and B are sets in some universe $\mathcal{U}$. The **symmetric difference** of A and B, denoted by $A \triangle B$, is defined as
$$A \triangle B := (A - B) \cup (B - A),$$
where $A - B$ denotes the set of elements in A that are not in B. That is, $A - B = \{x \in A \mid x \notin B\}$. We can also write the symmetric difference in the form
$$A \triangle B = (A \cup B) - (A \cap B).$$
The **power set** of a set S, denoted by $\mathscr{P}(S)$, is the set of all subsets of S.

   Let S be any set.
   (a) Show that $\triangle$ is a binary operation on $\mathscr{P}(S)$.
   (b) Show that $(\mathscr{P}(S), \triangle)$ is a group.
   (c) Show that $(\mathscr{P}(S), \triangle)$ is abelian.
   (d) Is $(\mathscr{P}(S), \cap)$ a group?
   (e) Is $(\mathscr{P}(S), \cup)$ a group?