

1. Suppose G is a group, and let $g \in G$. Let $\varphi_g : G \rightarrow G$ be a function defined by $\varphi_g(x) = gxg^{-1}$. Show that φ_g is an automorphism of G .
2. Let G be an abelian group. Let $f : G \rightarrow G$ be a function defined by $f(x) = x^{-1}$.
 - (a) Show that f is an automorphism of G .
 - (b) Show that if $G = S_3$, then f is not an automorphism.
 - (c) Show that if G is non-abelian, then f is not an automorphism.
3. Let $\varphi : G \rightarrow G$ be an automorphism. Let $H := \{x \in G \mid \varphi(x) = x\}$. Show that $H \leq G$.