(h) $g(x) = (1-x)^e + \int_5^x \frac{e^t}{1-t}dt$

$$g'(x) = e(1-x)^{e-1}\cdot(-1) + \frac{e^x}{1-x}$$

(i) $g(x) = \int_{-3x}^{e^{5x}} \frac{\cos t}{t^2+9}dt$ $= \int_{-3x}^{a} \frac{\cos t}{t^2+9}dt + \int_{a}^{e^{5x}} \frac{\cos t}{t^2+9}dt = -\int_{a}^{-3x} \frac{\cos t}{t^2+9}dt + \int_{a}^{e^{5x}} \frac{\cos t}{t^2+9}dt$

$$\Rightarrow g'(t) = -\frac{\cos(-3x)}{(-3x)^2+9}\cdot(-3) + \frac{\cos(e^{5x})}{(e^{5x})^2+9}\cdot 5e^{5x}$$

(j) $y = \ln\left[\frac{e^{3x}(x^2+7)^5\sqrt{1-x}}{(4-3x)^7}\right]$ $= \ln(e^{3x}) + \ln(x^2+7)^5 + \ln(1-x)^{1/2} - \ln(4-3x)^7$

$$= 3x + 5\ln(x^2+7) + \frac{1}{2}\ln(1-x) - 7\ln(4-3x)$$

$$y' = 3 + \frac{5(2x)}{x^2+7} + \frac{-1}{2(1-x)} - \frac{7(-3)}{4-3x}$$

(k) $y = \cosh(\ln x)$

$$y' = \sinh(\ln x)\cdot\frac{1}{x}$$

(l) $y = \left[\sinh(x^2)\right]^3$

$$y' = 3\left[\sinh(x^2)\right]^2\cdot\cosh(x^2)\cdot(2x)$$

(m) $y = \ln(\cosh x)$

$$y' = \frac{1}{\cosh x} \cdot \sinh x = \tanh x$$

(n) $y = \sinh(\sqrt{2})$

$y' = 0$. Since $\sinh(\sqrt{2})$ is just a number.

9. Use logarithmic differentiation to find the derivative of $y = (\tan x)^{\ln x}$.

$$\Rightarrow \ln y = \ln\left((\tan x)^{\ln x}\right)$$

$$\ln y = \ln x \cdot \ln(\tan x)$$

$$\frac{1}{y} \cdot y' = \ln x \cdot \frac{\sec^2 x}{\tan x} + \ln(\tan x) \cdot \frac{1}{x}$$

$$y' = (\tan x)^{\ln x} \cdot \left(\ln x \cdot \frac{\sec^2 x}{\tan x} + \frac{\ln(\tan x)}{x}\right)$$

10. Given $x^2 + xy^4 - y^{\sqrt{2}} = e^{2y}$, find $\frac{dy}{dx}$.

☆

diff wrt $x \Rightarrow$

$$2x + x \cdot 4y^3 \cdot \frac{dy}{dx} + y^4 - \sqrt{2}y^{\sqrt{2}-1} \cdot \frac{dy}{dx} = 2e^{2y} \cdot \frac{dy}{dx}$$

$$2x + y^4 = 2e^{2y}\frac{dy}{dx} - 4xy^3\frac{dy}{dx} + \sqrt{2}y^{\sqrt{2}-1}\frac{dy}{dx}$$

$$\frac{2x + y^4}{2e^{2y} - 4xy^3 + \sqrt{2}y^{\sqrt{2}-1}} = \frac{dy}{dx}$$

11. Given $\cos(xy) + \sin(x+y) = 1$, find $\frac{dy}{dx}$.

$$-\sin(xy)\cdot\left(x\frac{dy}{dx}+y\right) + \cos(x+y)\cdot\left(1+\frac{dy}{dx}\right) = 0$$

$$-x\sin(xy)\cdot\frac{dy}{dx} - y\sin(xy) + \cos(x+y) + \cos(x+y)\cdot\frac{dy}{dx} = 0$$

$$\cos(x+y)\frac{dy}{dx} - x\sin(xy)\frac{dy}{dx} = y\sin(xy) - \cos(x+y)$$

$$\frac{dy}{dx}\left[\cos(x+y) - x\sin(xy)\right] = y\sin(xy) - \cos(x+y)$$

$$\frac{dy}{dx} = \frac{y\sin(xy) - \cos(x+y)}{\cos(x+y) - x\sin(xy)}$$

12. Take the second derivative of $y = \frac{2}{5x+1}$. Simplify your answer completely.

$y = 2(5x+1)^{-1}$

☆ $y' = -2(5x+1)^{-2}\cdot 5$

$y' = -10(5x+1)^{-2}$

$y'' = 20(5x+1)^{-3}\cdot 5$

$$y'' = \frac{100}{(5x+1)^3}$$

13. Find the equation of the tangent line to $y = \sin^2 x$ at $x = \frac{\pi}{3}$.

@ $x = \frac{\pi}{3}$, $y = \sin^2\frac{\pi}{3} = \left(\frac{\sqrt{3}}{2}\right)^2 = 3/4$

$\Rightarrow$ $y = 3/4$

$y' = 2\sin x\cos x$

$y'\left(\frac{\pi}{3}\right) = 2\sin\frac{\pi}{3}\cos\frac{\pi}{3}$

$m = 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

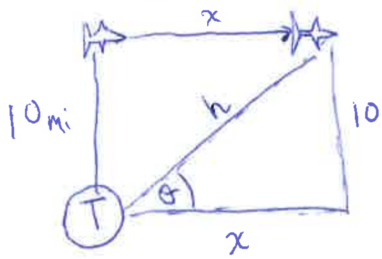
$m = \frac{\sqrt{3}}{2}$

$\Rightarrow$ $y - y_1 = m(x - x_1)$

$y - 3/4 = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right)$

$$y = \frac{\sqrt{3}}{2}x - \frac{\pi\sqrt{3}}{6} + \frac{3}{4}$$

14. A plane is flies horizontally at an altitude of 10 miles and passes directly over a tracking telescope on the ground. When the angle of elevation is $\frac{\pi}{4}$, this angle is decreasing at a rate of $\frac{\pi}{6}$ radians per minute. How fast is the plane traveling when the angle of elevation is $\frac{\pi}{4}$?



Let θ = the angle of elevation

x = the distance the plane has travelled since it was directly over the telescope.

We know

$\frac{d\theta}{dt} = \frac{-\pi}{6}$ (note: this is a negative rate since the angle of elevation decreases over time.)

$\theta = \frac{\pi}{4}$

$x = 10$ and $h = 10\sqrt{2}$ (from basic trig)

We want

$\frac{dx}{dt} = ?$

$$\tan\theta = \frac{10}{x}$$

$$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}(10x^{-1})$$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = -10x^{-2} \cdot \frac{dx}{dt}$$

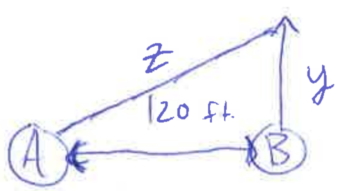
$$\frac{1}{\cos^2\theta} \cdot \left(-\frac{\pi}{6}\right) = -10(10)^{-2} \cdot \frac{dx}{dt}$$

$$\frac{1}{\left(\frac{10}{10\sqrt{2}}\right)} \cdot \left(\frac{-\pi}{6}\right) = \frac{-10}{100} \cdot \frac{dx}{dt}$$

$$\frac{-\pi\sqrt{2}}{6} = \frac{-1}{10}\frac{dx}{dt}$$

$$\Rightarrow \boxed{\frac{dx}{dt} = \frac{5\pi\sqrt{2}}{3} \text{ miles per minute}}$$

15. Albert stands 120 ft to the west of Bill. Bill starts running to the north at a speed of 10 ft/sec. How fast is the distance between Albert and Bill is increasing at the end of 5 seconds?



We know

$\frac{dy}{dt} = 10$

at 5 sec

$y = 5 \cdot 10 = 50$ ft

$z = 130$ ft. (from pythagorean theorem)

We want

$\frac{dz}{dt} = ?$

$$120^2 + y^2 = z^2$$

$$\frac{d}{dt}(120^2) + \frac{d}{dt}(y^2) = \frac{d}{dt}(z^2)$$

$$2y \cdot \frac{dy}{dt} = 2z \cdot \frac{dz}{dt}$$

$$2(50)(10) = 2(130) \cdot \frac{dz}{dt}$$

$$1000 = 260\frac{dz}{dt}$$

$$\Rightarrow \frac{dz}{dt} = \frac{1000}{260} = \boxed{\frac{50}{13} \text{ ft/sec}}$$

16. Given $f(x) = x^3 - 6x^2 - 15x + 1$,

(a) Find all critical numbers for $f(x)$ on the interval $(-\infty, \infty)$.

$f'(x) = 3x^2 - 12x - 15$

$f'(x) = 3(x^2 - 4x - 5) = 0$

$3(x-5)(x+1) = 0$

$\boxed{\text{C.N.'s } \; x = 5, \; x = -1}$

(b) Find the absolute maximum and absolute minimum of $f(x)$ on the interval $[-2, 1]$.

Only C.N. in the interval $[-2,1]$ is $x = -1$.

$f(-1) = (-1)^3 - 6(-1)^2 - 15(-1) + 1 = \boxed{9}$

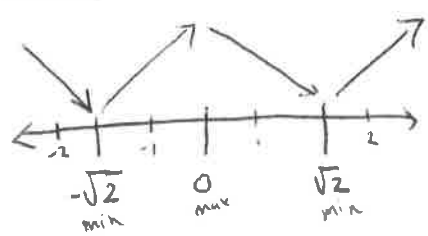
$f(-2) = (-2)^3 - 6(-2)^2 - 15(-2) + 1 = -1$

$f(1) = (1)^3 - 6(1)^2 - 15(1) + 1 = \boxed{-19}$

$\Rightarrow$ $\boxed{\text{absolute max: } (-1, 9) \quad \text{absolute min: } (1, -19)}$

17. Find the local extrema of the following functions.

(a) $f(x) = x^4 - 4x^2 + 12$



$f'(x) = 4x^3 - 8x$

$= 4x(x^2 - 2) = 0$

$4x = 0 \quad | \quad x^2 - 2 = 0$

C.N.'s $\quad x = 0 \quad | \quad x^2 = 2$

$x = \pm\sqrt{2}$

$\boxed{\text{local mins @ } (-\sqrt{2}, 8) \text{ and } (\sqrt{2}, 8) \quad \text{local max @ } (0, 12)}$

$f'(-2) = 4(-2)((-2)^2 - 2) = -8(2) < 0 \Rightarrow$ decreasing

$f'(-1) = 4(-1)((-1)^2 - 2) = -4(-1) > 0 \Rightarrow$ increasing

$f'(1) = 4(1)((1)^2 - 2) = 4(-1) < 0 \Rightarrow$ dec.

$f'(2) = 4(2)((2)^2 - 2) = 8(2) > 0 \Rightarrow$ inc.

(b) $g(x) = xe^{2x}$

$g'(x) = x \cdot 2e^{2x} + e^{2x}$

$g'(x) = e^{2x}(2x+1) = 0$

$e^{2x} = 0 \quad | \quad 2x + 1 = 0$

No soln. $\quad | \quad x = \frac{-1}{2}$

C.N.

$g''(x) = e^{2x}(2) + (2x+1) \cdot 2e^{2x}$

$g''(x) = 2e^{2x}(1 + (2x+1))$

11 $g''(x) = 2e^{2x}(2x+2)$

$g''(-\frac{1}{2}) = 2e^{2(-\frac{1}{2})}(2(-\frac{1}{2}) + 2) = 2e^{-1}(1) > 0 \Rightarrow \cup \Rightarrow$ local min

$\boxed{\text{local min @ } (-\tfrac{1}{2}, -\tfrac{1}{2e})}$

$\boxed{\text{V.A. at } x=2,\ x=-2}$

$$\lim_{x\to\infty} \frac{x^2}{x^2-4} = \lim_{x\to\infty} \frac{1}{1-\frac{4}{x^2}} = 1 \Rightarrow \boxed{\text{H.A. @ } y=1}$$

($\frac{4}{x^2} \to 0$)

18. Use the methods of Section 4.5 to find for f: the domain, intercepts, symmetry, asymptotes, intervals where f is increasing/decreasing, local extrema, intervals where f is concave up/down, and any inflection points. Use this information to graph f.

(a) $f(x) = \dfrac{x^2}{x^2-4}$

$$f(-x) = \frac{(-x)^2}{(-x)^2-4} = \frac{x^2}{x^2-4} = f(x) \Rightarrow \text{even fcn}$$

Domain: $\{x \mid x \neq \pm 2\}$

x-intercept: $(0,0)$

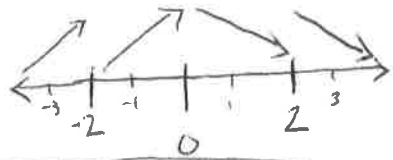
y-intercept: $(0,0)$

Symmetry: even fcn, symmetric over the y-axis

$$f'(x) = \frac{(x^2-4)2x-(x^2)(2x)}{(x^2-4)^2} = \frac{2x(\cancel{x^2}-4-\cancel{x^2})}{(x^2-4)^2} = \frac{-8x}{(x^2-4)^2}$$

C.N's where $f'(x)$ is undefined or $f'(x)=0$

$\Rightarrow$ C.N's $x=\pm 2$, $x=0$



$f'(-3) = \dfrac{-8(-3)}{\text{positive}} > 0 \Rightarrow$ increasing

$f'(-1) = \dfrac{-8(-1)}{+} > 0 \Rightarrow$ increasing

$f'(1) = \dfrac{-8(1)}{+} < 0 \Rightarrow$ decreasing

$f'(3) = \dfrac{-8(3)}{+} < 0 \Rightarrow$ dec

f is increasing on $(-\infty,-2)\cup(-2,0)$

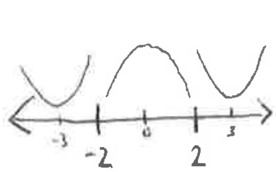
f is decreasing on $(0,2)\cup(2,\infty)$

local max @ $(0,0)$ | no local min

f is concave up on $(-2,2)$

f is concave down $(-\infty,-2)$, $(2,\infty)$

No inflection pts. since f is discontinuous @ $x=\pm 2$



$f''(-3) = \frac{248}{125} > 0$

$f''(0) = -\frac{1}{2} < 0$

$f''(3) = \ > 0$

$$f''(x) = \frac{(x^2-4)^2(-8)-(-8x)(2)(x^2-4)(2x)}{(x^2-4)^4}$$

$$f''(x) = \frac{\cancel{(x^2-4)}\left[-8x^2+32+32x^2\right]}{(x^2-4)^{\cancel{4}\,3}}$$

$$f''(x) = \frac{-8\left[x^2-4-4x^2\right]}{(x^2-4)^3}$$

$$f''(x) = \frac{-8(-3x^2-4)}{(x^2-4)^3}$$

concavity of f may change at $x=\pm 2$, or $-3x^2-4=0$

NO SOLN