(b) $f(x) = \dfrac{x}{x^2+4}$

$\boxed{\text{Domain: all reals}}$

$\boxed{x\text{-int: } (0,0)}$

$\boxed{y\text{-int: } (0,0)}$

$\boxed{\text{symmetry: odd fcn, symmetric about the origin}}$

$\boxed{\text{No V.A.'s}}$

$\boxed{\text{H.A.: } y=0}$

$\boxed{f \text{ is increasing on } (-2,2)}$

$\boxed{f \text{ is decreasing on } (-\infty,-2), (2,\infty)}$

$\boxed{\text{local max at } (2, 1/4)}$

$\boxed{\text{local min at } (-2, -1/4)}$

$\boxed{f \text{ is concave up: } (-2\sqrt{3}, 0), (2\sqrt{3}, \infty)}$

$\boxed{f \text{ is concave down: } (-\infty, -2\sqrt{3}), (0, 2\sqrt{3})}$

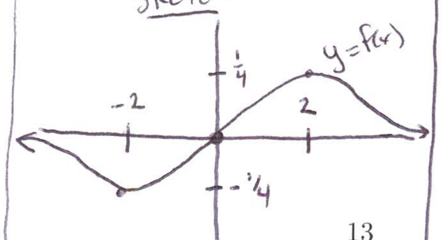
Inflection points:

$\left(-2\sqrt{3}, \frac{-\sqrt{3}}{8}\right)$

$(0,0)$

$\left(2\sqrt{3}, \frac{\sqrt{3}}{8}\right)$

sketch

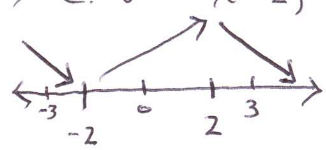


$$f(-x) = \frac{-x}{(-x)^2+4} = \frac{-x}{x^2+4} = -f(x) \Rightarrow \text{odd fcn.}$$

$$\lim_{x\to\infty} \frac{x}{x^2+4} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1+\frac{4}{x^2}} = 0 \Rightarrow \text{H.A. at } y=0.$$

$$f'(x) = \frac{(x^2+4)(1) - x(2x)}{(x^2+4)^2} = \frac{x^2+4-2x^2}{(x^2+4)^2} = \frac{4-x^2}{(x^2+4)^2} = 0$$

$\Rightarrow$ C.N.'s $x=2$, $x=-2$



$f'(-3) < 0 \Rightarrow$ dec.

$f'(0) > 0 \Rightarrow$ inc

$f'(3) < 0 \Rightarrow$ dec.

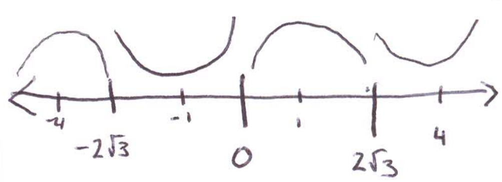
$$f''(x) = \frac{(x^2+4)^{\cancel{2}}(-2x) - (4-x^2)(2)\cancel{(x^2+4)}(2x)}{(x^2+4)^{\cancel{4}\,3}}$$

$$f''(x) = \frac{2x\left(-x^2-4-((4-x^2)(2))\right)}{(x^2+4)^3}$$

$$f''(x) = \frac{2x(-x^2-4-8+2x^2)}{(x^2+4)^3} = \frac{2x(x^2-12)}{(x^2+4)^3}$$

Concavity may change at

$2x(x^2-12)=0 \Rightarrow x=0,\ x=\pm\sqrt{12}=\pm 2\sqrt{3}$



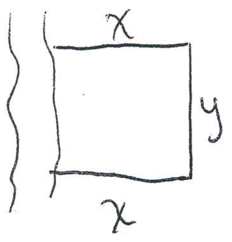
$f''(-4) = -8(4) < 0 \Rightarrow$ concave ↓

$f''(-1) = -2(-11) > 0 \Rightarrow$ concave ↑

$f''(1) = 2(-11) < 0 \Rightarrow$ concave ↓

$f''(4) = 8(4) > 0 \Rightarrow$ concave ↑

19. A farmer wants to fence in a field with area 800 square feet. One side of the field will border a river and does not require fencing. What is the minimum amount of fencing needed? Label your answer and show a check that you have found a minimum.



$A = 800$

$xy = 800$

$y = \frac{800}{x}$

$f = 2x + y$

$f(x) = 2x + 800x^{-1}$

$f'(x) = 2 - 800x^{-2} = 0$

$\Rightarrow 2 = \frac{800}{x^2}$

$\Rightarrow 2x^2 = 800$

$\Rightarrow x^2 = 400$

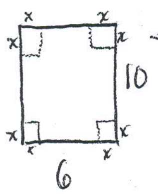
$x = 20$

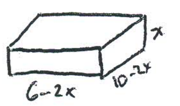
$y = \frac{800}{20} = 40$

$f''(x) = \frac{1600}{x^3}$; $f''(20) = \frac{1600}{20^3} > 0 \Rightarrow \cup \Rightarrow$ min ✓

Thus, the minimum amount of fencing is $f = 2(20) + 40$
$= \boxed{80 \text{ ft}}$

20. An open box is made from a rectangle piece of paper with 10 cm in length and by 6 cm in width, by cuttingg equal squares from each corner and folding up the sides. Make a careful sketch and find the volume of the box with the greatest capacity that can be so constructed.





length = 6-2x
width = 10-2x
height = x

Volume
$V(x) = (6-2x)(10-2x)(x)$

$V(x) = 4x^3 - 32x^2 + 60x$

$V'(x) = 12x^2 - 64x + 60 = 0$

use quad. form.

$V'(x) = 2(6x^2 - 32x + 30)$

$x = \frac{+32 \pm \sqrt{(-32)^2 - 4(6)(30)}}{2(6)} = \frac{8 \pm \sqrt{19}}{3}$

$x \approx 1.21, \cancel{4.12}$

We can "throw out" when $x = \frac{8+\sqrt{19}}{3} \approx 4.12$ since if we cut out 2 squares of size ≈ 4.12, then our cuts would equal $\approx 8.24 > 6$, which is physically impossible.

$V''(x) = 24x - 64$

$V''\left(\frac{8-\sqrt{19}}{3}\right) = 24\left(\frac{8-\sqrt{19}}{3}\right) - 64$

$= 8(8-\sqrt{19}) - 64$

$= \cancel{64} - 8\sqrt{19} - \cancel{64}$

$= -8\sqrt{19} < 0 \Rightarrow \cap$

$\Rightarrow$ max ✓

Thus, max volume $\approx \boxed{32.8 \text{ cm}^3}$

21. State the Mean Value Theorem. If f is a function that is continuous on $[a,b]$, and differentiable on (a,b), then there is a number c in (a,b) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

22. Find the most general antiderivative of $f(x) = 4\sec^2 x - \sec x \tan x + 3e^x$

$$F(x) = 4\tan x - \sec x + 3e^x + C$$

23. Find f given $f'(x) = 8x^3 + \dfrac{3}{x} + \dfrac{2}{x^2} + 1$ and $f(1) = 7$.

$$f'(x) = 8x^3 + 3x^{-1} + 2x^{-2} + 1$$

$$f(x) = \frac{8x^4}{4} + 3\ln|x| + \frac{2x^{-1}}{-1} + x + C$$

$$\Rightarrow \boxed{f(x) = 2x^4 + 3\ln|x| - \frac{2}{x} + x + 6}$$

$$f(x) = 2x^4 + 3\ln|x| - \frac{2}{x} + x + C$$

$$f(1) = 2(1)^4 + 3\ln|1| - \frac{2}{1} + 1 + C$$

$$7 = 2 + 0 - 2 + 1 + C$$

$$6 = C$$

24. Evaluate the following definite integrals.

(a) $\displaystyle\int_3^3 x^2 \sin 4x dx = 0$

(b) $\displaystyle\int_1^8 \sqrt{3x+1}dx$

let $u = 3x+1 \Rightarrow u$ bounds $\to (4, 25)$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$$= \frac{1}{3}\int_4^{25} u^{1/2}\,du$$

$$= \frac{1}{3}\left.\frac{u^{3/2}}{(3/2)}\right]_4^{25} = \frac{2}{9}\left(25^{3/2} - 4^{3/2}\right) = \frac{2}{9}\cdot(125-8) = \frac{234}{9} = 26$$

(c) $\int_0^3 \frac{e^{3x}}{e^{3x}-5}dx$ = DNE

since the integrand is undefined when $e^{3x}-5=0 \Rightarrow$ at $x=\frac{\ln 5}{4} \approx .54$

which is in $(0,3)$.

(d) $\int_0^{\frac{\pi}{4}} e^{\tan x}\sec^2 x dx$

let $u=\tan x$ $\Rightarrow$ u-bounds: $(0,1)$

$du=\sec^2 x dx$

$= \int_0^1 e^u du$

$= e^u\Big]_0^1 = e^1 - e^0 = e-1$

25. Evaluate the following indefinite integrals.

(a) $\int (\sqrt[3]{x} - 4 + e^x)dx$

$= \frac{3x^{4/3}}{4} - 4x + e^x + C$

(b) $\int \sin 4x dx$

let $u=4x$

$du = 4dx$

$\frac{1}{4}du = dx$

$= \frac{1}{4}\int \sin u du$

$= \frac{1}{4}[-\cos u] + C$

$= -\frac{1}{4}\cos 4x + C$

(c) $\int \frac{1}{1+(5x)^2}dx$

let $u=5x$

$du = 5dx$

$\frac{1}{5}du = dx$

$= \frac{1}{5}\int \frac{1}{1+u^2}du$

$= \frac{1}{5}\left[\tan^{-1} u\right] + C$

$= \frac{1}{5}\tan^{-1}(5x) + C$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$u-1 = x^2$

$= \frac{1}{2}\int (u-1)\sqrt{u}\,du$

$= \frac{1}{2}\int u^{3/2} - u^{1/2}\,du$

$= \frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C$

(e) $\displaystyle\int \left(\frac{1}{x} - \frac{1}{x^3} + \sqrt[3]{x} - x^e - 3^{\sqrt{5}} + \csc x \cot x\right) dx$

$= \ln|x| + \frac{1}{2x^2} + \frac{3x^{4/3}}{4} - \frac{x^{e+1}}{e+1} - 3^{\sqrt{5}}x - \csc x + C$

(f) $\displaystyle\int (\sinh t + \cosh t)\, dt$

$= \cosh t + \sinh t + C$