Final Review MTH 201

1. An object whose position (in feet) at time t (where t is in seconds) is given by $s(t) = 2t^2 + 5t + 1$, (label your answers appropriately)

   (a) Find the average velocity of the object on the interval $[0, 2]$.

   (b) Find the instantaneous velocity of the object at $t = 1$.

2. Determine the following limits:

   (a) $\displaystyle\lim_{x\to 2} \frac{x^2 + x - 6}{x^2 - 7x + 10}$

   (b) $\displaystyle\lim_{x\to \infty} \frac{5x^3 - 1}{x + x^2 - 2x^3}$

   (c) $\displaystyle\lim_{x\to -\infty} \frac{500x^2 + 750x + 1000}{1 - 3x^3}$

   (d) $\displaystyle\lim_{x\to -\infty} \frac{1 - x^3}{x^2 + x + 1000}$

(e) $\lim_{x \to \frac{\pi}{2}} \cos(2x + \cos x)$

(f) $\lim_{x \to 2^-} \frac{x^2 - 4}{|x - 2|}$

(g) $\lim_{x \to 0} \left(\frac{1 - \cos 2x}{x} + \frac{\sin 3x}{x} \right)$

(h) $\lim_{x \to 6^-} [x]$ Please see EXAMPLE 10 (p.105) for the definition of this function. This function is sometimes called the greatest integer function (or floor function).

(i) $\lim_{x \to 6} [x]$

(j) $\lim_{x \to 6^+} ([x] + 2x)$

(k) $\lim_{x \to 1} \frac{\ln x}{x - 1}$

(l) $\lim_{x\to\infty} 5x^2e^{-x}$

(m) $\lim_{x\to 0} \frac{e^x - 1 - x}{x^2}$

(n) $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$.

3. Use the Squeeze Theorem to show that $\lim_{x\to 0} x^6 \cos(\ln|x|) = 0$

4. What are the vertical and horizontal asymptotes of the function $f(x) = \dfrac{2x^2 - 6x}{x^2 - 9}$? Label the asymptotes as to whether they are vertical or horizontal. Be sure to clearly and completely justify your answers.

5. What is the definition of the derivative of $f(x)$?

6. Find the derivative of $g(x) = \frac{4}{2x-3}$ using the definition of the derivative.

7. Find the derivative of $g(x) = x^2 + 3x$ using the definition of the derivative.

8. Differentiate.

(a) $f(x) = 3x^5 - 4x^2 + x - 2 + 6e^x + 5^e$

(b) $h(\theta) = \tan(\sin(7\theta + 1))$

(c) $y = x\tan^{-1}(3x) - \sin^{-1}(3x) + [\sin 3x]^{-1}$

(d) $y = [3\ln x + \cot x]^{\ln 4} - \tan^7[5x + \pi^x]^2$

(e) $h(x) = 1 - \dfrac{7}{\sqrt{x}} + \dfrac{2}{3x+5} - 3\sqrt{2x-1} + \dfrac{5}{6x}$

(f) $g(x) = \dfrac{\cos 4x}{3 - \sin 4x}$

(g) $y = e^{x^3+x^2+x+1} + \ln(2 - 5x + 7x^3) - \pi^3$

(h) $g(x) = (1-x)^e + \int_5^x \frac{e^t}{1-t} dt$

(i) $g(x) = \int_{-3x}^{e^{5x}} \frac{\cos t}{t^2+9} dt$

(j) $y = \ln\left[\frac{e^{3x}(x^2+7)^5\sqrt{1-x}}{(4-3x)^7}\right]$

(k) $y = \cosh(\ln x)$

(l) $y = \left[\sinh(x^2)\right]^3$

(m) $y = \ln(\cosh x)$

(n) $y = \sinh(\sqrt{2})$

9. Use logarithmic differentiation to find the derivative of $y = (\tan x)^{\ln x}$.

10. Given $x^2 + xy^4 - y^{\sqrt{2}} = e^{2y}$, find $\frac{dy}{dx}$.

11. Given $\cos xy + \sin(x+y) = 1$, find $\frac{dy}{dx}$.

12. Take the second derivative of $y = \frac{2}{5x+1}$. Simplify your answer completely.

13. Find the equation of the tangent line to $y = \sin^2 x$ at $x = \frac{\pi}{3}$.

14. A plane is flies horizontally at an altitude of 10 miles and passes directly over a tracking telescope on the ground. When the angle of elevation is $\frac{\pi}{4}$, this angle is decreasing at a rate of $\frac{\pi}{6}$ radians per minute. How fast is the plane traveling?

15. Albert stands 120 ft to the west of Bill. Bell starts running to the north at a speed of 10 ft/sec. How fast is the distance between Albert and Bill is increasing at the end of 5 seconds?

16. Given $f(x) = x^3 - 6x^2 - 15x + 1$,

   (a) Find all critical numbers for $f(x)$ on the interval $(-\infty, \infty)$.

   (b) Find the absolute maximum and absolute minimum of $f(x)$ on the interval $[-2, 1]$.

17. Find the local extrema of the following functions.

   (a) $f(x) = x^4 - 4x^2 + 12$

   (b) $g(x) = xe^{2x}$

18. Use the methods of Section 4.5 to find for f: the domain, intercepts, symmetry, asymptotes, intervals where f is increasing/decreasing, local extrema, intervals where f is concave up/down, and any inflection points. Use this information to graph f.

   (a) $f(x) = \dfrac{x^2}{x^2 - 4}$

(b) $f(x) = \dfrac{x}{x^2 + 4}$

19. A farmer wants to fence in a field with area 800 square feet. One side of the field will border a river and does not require fencing. What is the minimum amount of fencing needed? Label your answer and show a check that you have found a minimum.

20. An open box is made from a rectangle piece of paper with 10 cm in length and by 6 cm in width, by cuttingg equal squares from each corner and folding up the sides. Make a careful sketch and find the volume of the box with the greatest capacity that can be so constructed.

21. State the Mean Value Theorem.

22. Find the most general antiderivative of $f(x) = 4\sec^2 x - \sec x \tan x + 3e^x$

23. Find f given $f'(x) = 8x^3 + \frac{3}{x} + \frac{2}{x^2} + 1$ and $f(1) = 7$.

24. Evaluate the following definite integrals.

(a) $\int_3^3 x^2 \sin 4x dx$

(b) $\int_1^8 \sqrt{3x+1} dx$

(c) $\displaystyle\int_0^3 \frac{e^{3x}}{e^{3x}-5}dx$

(d) $\displaystyle\int_0^{\frac{\pi}{4}} e^{\tan x}\sec^2 x dx$

25. Evaluate the following indefinite integrals.

(a) $\displaystyle\int (\sqrt[3]{x} - 4 + e^x)dx$

(b) $\displaystyle\int \sin 4x dx$

(c) $\displaystyle\int \frac{1}{1+(5x)^2}dx$

(d) $\int x^3\sqrt{x^2+1}dx$

(e) $\int \left(\frac{1}{x} - \frac{1}{x^3} + \sqrt[3]{x} - x^e - 3^{\sqrt{5}} + \csc x \cot x\right) dx$

(f) $\int (\sinh t + \cosh t)\, dt$