EXAM 2

Score: ________ out of 100

Math 201 - Calculus I **Name:** key

Read all of the following information before starting the exam:

- You have 60 minutes to complete the exam.
- Show all work, clearly and in order, if you want to get full credit. Please make sure you read the directions for each problem. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Please $\boxed{\text{box/circle}}$ or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 5 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. Use the limit definition of the derivative to differentiate $f(x) = \sqrt{x+2}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\left(\frac{\sqrt{x+h+2}-\sqrt{x+2}}{h}\right)\left(\frac{\sqrt{x+h+2}+\sqrt{x+2}}{\sqrt{x+h+2}+\sqrt{x+2}}\right)$$

$$= \lim_{h\to 0} \frac{\cancel{x}+h+\cancel{2}-(\cancel{x+2})}{h(\sqrt{x+h+2}+\sqrt{x+2})}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h+2}+\sqrt{x+2})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h+2}+\sqrt{x+2}}$$

$$= \frac{1}{\sqrt{x+2}+\sqrt{x+2}} = \boxed{\frac{1}{2\sqrt{x+2}}}$$

answer: $f'(x) = \dfrac{1}{2\sqrt{x+2}}$

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2. Find f' in terms of g' for each of the following (full simplification is not necessary):

(a) $f(x) = x^2 g(x)$

$$f'(x) = x^2 g'(x) + \frac{d}{dx}(x^2)\, g(x)$$

answer: $f'(x) = x^2 g'(x) + 2x\, g(x)$

(b) $f(x) = e^{g(x)}$

$$f'(x) = e^{g(x)}\left(\frac{d}{dx} g(x)\right)$$

answer: $f'(x) = e^{g(x)} \cdot g'(x)$

(c) $f(x) = \sqrt{\frac{g(x)}{x}} = \left(\frac{g(x)}{x}\right)^{1/2}$

$$f'(x) = \frac{1}{2}\left(\frac{g(x)}{x}\right)^{-1/2} \frac{d}{dx}\left(\frac{g(x)}{x}\right)$$

answer: $f'(x) = \frac{1}{2}\left(\frac{g(x)}{x}\right)^{-1/2}\left(\frac{x g'(x) - g(x)}{x^2}\right)$

3. Compute the derivative for each of the following. Full simplification is not necessary, but your final answer should not have any derivatives and all answers should be functions in the appropriate variable.

(a) $y = 3x^2 + \log_3(x) - 5^x + \sqrt[3]{x}$ $= 3x^2 + \log_3(x) - 5^x + x^{1/3}$

answer: $y' = 6x + \frac{1}{\ln(3)x} - \ln(5)5^x + \frac{1}{3}x^{-2/3}$

(b) $y = \sqrt{\sinh(x) + \sin^{-1}(x)}$

$= \left(\sinh(x) + \sin^{-1}(x)\right)^{1/2}$

answer: $y' = \frac{1}{2}\left(\sinh(x) + \sin^{-1}(x)\right)^{-1/2}\left(\cosh(x) + \frac{1}{\sqrt{1-x^2}}\right)$

(c) $y = \ln(x)\sin^3(4x^2 + 7)$ $= \ln(x)\left(\sin(4x^2+7)\right)^3$

$y' = \ln(x)\frac{d}{dx}\left(\sin(4x^2+7)\right)^3 + \left(\sin(4x^2+7)\right)^3\frac{d}{dx}\left(\ln(x)\right)$

answer: $y' = \ln(x)\,3\sin^2(4x^2+7)\cos(4x^2+7)(8x) + \sin^3(4x^2+7)\left(\frac{1}{x}\right)$

(d) $y = \dfrac{2 - e^x}{1 + \sec(x)}$

$$y' = \frac{(1+\sec(x))\frac{d}{dx}(2-e^x) - (2-e^x)\frac{d}{dx}(1+\sec(x))}{(1+\sec(x))^2}$$

answer: $\boxed{y' = \dfrac{(1+\sec(x))(-e^x) - (2-e^x)(\sec(x)\tan(x))}{(1+\sec(x))^2}}$

(e) $y = \ln\Big(\tan(2x^5 - 1)\Big)$

$$y' = \frac{1}{\tan(2x^5-1)} \frac{d}{dx} \tan(2x^5-1)$$

answer: $\boxed{y' = \dfrac{1}{\tan(2x^5-1)} \sec^2(2x^5-1)(10x^4)}$

(f) $y = (1 - x)^{\cosh(x)}$

$$\ln y = \ln (1-x)^{\cosh(x)}$$

$$\ln y = \cosh(x) \ln(1-x)$$

$$\frac{d}{dx} \ln y = \frac{d}{dx}\left(\cosh(x) \ln(1-x)\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \cosh(x)\left(\frac{1}{1-x}(-1)\right) + \ln(1-x)\sinh(x)$$

answer: $\boxed{\dfrac{dy}{dx} = (1-x)^{\cosh(x)}\left(\cosh(x)\left(\dfrac{-1}{1-x}\right) + \ln(1-x)\sinh(x)\right)}$

4. Consider the implicit equation

$$e^{x-y} = \cos(xy)$$

(a) Use implicit differentiation to find $\frac{dy}{dx}$

$$\frac{d}{dx} e^{x-y} = \frac{d}{dx} \cos(xy)$$

$$e^{x-y} \frac{d}{dx}(x-y) = -\sin(xy) \frac{d}{dx}(xy)$$

$$e^{x-y}\left(1 - \frac{dy}{dx}\right) = -\sin(xy)\left(x \frac{dy}{dx} + y\right)$$

$$e^{x-y} - e^{x-y}\frac{dy}{dx} = -\sin(xy)\, x \frac{dy}{dx} - \sin(xy)\, y$$

$$\frac{dy}{dx}\left(-e^{x-y} + \sin(xy)x\right) = -e^{x-y} - \sin(xy)\,y$$

answer: $\boxed{\frac{dy}{dx} = \frac{-e^{x-y} - \sin(xy)y}{-e^{x-y} + \sin(xy)x}}$

(b) Find an equation of the tangent line to the curve at the point $(\sqrt{2\pi}, \sqrt{2\pi})$.To **receive full credit you must simplify (I promise the answer is nice).**

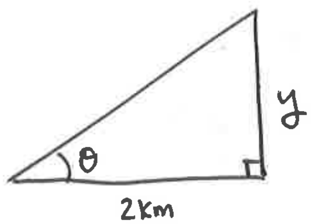
$$\text{Slope} = m = \left.\frac{dy}{dx}\right|_{(x,y)=(\sqrt{2\pi},\sqrt{2\pi})} = \frac{-e^{\sqrt{2\pi}-\sqrt{2\pi}} - \sin(2\pi)\sqrt{2\pi}}{-e^{\sqrt{2\pi}-\sqrt{2\pi}} + \sin(2\pi)\sqrt{2\pi}}$$

$$= \frac{-e^0 - 0}{-e^0 + 0} = \frac{-1}{-1} = 1$$

using the point slope formula: $y - \sqrt{2\pi} = 1(x - \sqrt{2\pi})$ OR

answer: $\boxed{y = x}$

5. You are watching a rocket launch exactly 2 km away. As the rocket climbs in altitude, you begin tilt your face upwards in order to follow it. If the rocket moves vertically upwards at a constant rate of 10 km/s, how fast is the angle at which you tilt your face changing when the rocket is 9 km above the launching station? (Note: Your picture should be something nice! Do not worry about meaningless things such as your height and the height of the rocket, etc.)



given: $\frac{dy}{dt} = 10 \frac{km}{s}$

unknown: $\frac{d\theta}{dt}$ when $y = 9$km.

Equation relating y and θ (many answers):

$$\tan\theta = \frac{y}{2}$$

Differentiate:

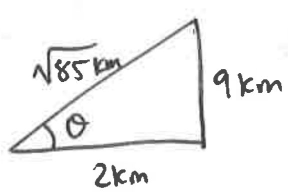
$$\frac{d}{dt}\tan\theta = \frac{d}{dt}\left(\frac{y}{2}\right)$$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{2}\frac{dy}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{2}\frac{dy}{dt}\frac{1}{\sec^2\theta} = \frac{1}{2\sec^2\theta}\frac{dy}{dt}$$

$$= \frac{\cos^2\theta}{2}\frac{dy}{dt}$$

Substitute:



$$\cos\theta = \frac{adj}{hyp} = \frac{2}{\sqrt{85}}$$

$$\frac{d\theta}{dt} = \frac{(2/\sqrt{85})^2}{2}(10) = \frac{4 \cdot 10}{2 \cdot 85} = \frac{20}{85}\frac{rad}{s}$$

answer: $\boxed{\frac{d\theta}{dt} = \frac{4}{17}\frac{rad}{s}}$