Score: ______ out of 10.

Math 201 - Fake (but useful) Quiz Name: key

1. Let $f(x) = \sqrt{x}$.

   (a) Use the definition of the dertivative to calculate $f'(4)$.

$$f'(4) = \lim_{h\to 0} \frac{\sqrt{4+h} - \sqrt{4}}{h} = \lim_{h\to 0} \frac{\sqrt{4+h} - 2}{h} \cdot \left(\frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}\right)$$

$$= \lim_{h\to 0} \frac{(\sqrt{4+h})^2 \cancel{-2\sqrt{4+h}} \cancel{+2\sqrt{4+h}} - 4}{h(\sqrt{4+h}+2)}$$

$$= \lim_{h\to 0} \frac{4+h-4}{h(\sqrt{4+h}+2)}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{4+h}+2)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{4+h}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \boxed{\frac{1}{4}}$$

   (b) Find an equation for the tangent line to the curve $f(x) = \sqrt{x}$ at $x = 4$.

Equation of tangent line (by point-slope formula):

$$y - f(a) = f'(a)(x-a)$$

So here since $a = 4$, $f(4) = \sqrt{4} = 2$, $f'(4) = 1/2$ (from part (a)):

$$\boxed{y - 2 = \frac{1}{4}(x-4)}$$

OR

$$y = \frac{1}{4}x - \frac{4}{4} + 2 = \frac{1}{4}x - 1 + 2 = \frac{1}{4}x + 1$$

$$\boxed{y = \frac{1}{4}x + 1}$$

2. Let $f(x) = \dfrac{2}{x+1}$. Use the definition of the derivative to find $f'(x)$.

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{2}{(x+h)+1} - \frac{2}{x+1}}{h} = \lim_{h\to 0} \frac{\frac{2}{x+h+1}\left(\frac{x+1}{x+1}\right) - \frac{2}{x+1}\left(\frac{x+h+1}{x+h+1}\right)}{h}$$

$$= \lim_{h\to 0} \frac{2x+2 - 2(x+h+1)}{h(x+h+1)(x+1)}$$

$$= \lim_{h\to 0} \frac{\cancel{2x} + \cancel{2} - \cancel{2x} - 2h - \cancel{2}}{h(x+h+1)(x+1)}$$

$$= \lim_{h\to 0} \frac{-2\cancel{h}}{\cancel{h}(x+h+1)(x+1)}$$

$$= \frac{-2}{(x+0+1)(x+1)} = \boxed{\frac{-2}{(x+1)^2}}$$