Score: _____ out of 10.

Math 201 - Quiz #4 Name: key

1. **Solve two of the following parts. Please put an X through the parts you do not want graded.** Compute the derivative for each of the following. Full simplification is not necessary, but your final answer should not have any derivatives.

(a) $f(x) = 3\cos(x) + \sin(3x^7 - 1)$

need chain rule

$$f'(x) = -3\sin(x) + \cos(3x^7-1)\frac{d}{dx}(3x^7-1)$$
$$= -3\sin(x) + \cos(3x^7-1)(21x^6)$$

answer: $-3\sin(x) + \cos(3x^7-1)(21x^6)$

(b) $y = \dfrac{\sec(5x)}{x-1}$

need quotient rule

need chain rule

$$y' = \frac{(x-1)\frac{d}{dx}\sec(5x) - \sec(5x)\frac{d}{dx}(x-1)}{(x-1)^2} = \frac{(x-1)\sec(5x)\tan(5x)\cdot 5 - \sec(5x)(1)}{(x-1)^2}$$

answer: $\dfrac{(x-1)\sec(5x)\tan(5x)\cdot 5 - \sec(5x)}{(x-1)^2}$

(c) $y = \sqrt{\tan(2x+1)} = (\tan(2x+1))^{1/2}$ ← need chain rule

$$y' = \frac{1}{2}(\tan(2x+1))^{-1/2}\frac{d}{dx}(\tan(2x+1))$$
$$= \frac{1}{2}\frac{1}{\sqrt{\tan(2x+1)}}\sec^2(2x+1)\frac{d}{dx}(2x+1)$$
$$= \frac{1}{2}\frac{1}{\sqrt{\tan(2x+1)}}\sec^2(2x+1)(2)$$

answer: $\dfrac{\sec^2(2x+1)}{\sqrt{\tan(2x+1)}}$

(d) $y = e^{x\cot(x)}$

← need chain rule

$$y' = e^{x\cot(x)}\frac{d}{dx}(x\cot(x))$$

product rule

$$y' = e^{x\cot(x)}\left(x(-\csc^2(x)) + \cot(x)\right)$$

answer: $e^{x\cot(x)}\left(-x\csc^2(x) + \cot(x)\right)$

2. Find $\frac{dy}{dx}$ of the eqation $x^2 + xy + y^2 = \sin^2(x)$ using implicit differentiation.

$$\frac{d}{dx}\left(x^2 + \underbrace{xy}_{\text{product rule}} + \underbrace{y^2}_{\text{chain rule}}\right) = \underbrace{\frac{d}{dx}\left(\sin(x)\right)^2}_{\text{chain rule (or product rule)}}$$

$$2x + \left(x\frac{dy}{dx} + y\right) + 2y\frac{dy}{dx} = 2\sin(x)\cos(x)$$

$$x\frac{dy}{dx} + 2y\frac{dy}{dx} = 2\sin(x)\cos(x) - 2x - y$$

$$\frac{dy}{dx}\left(x + 2y\right) = 2\sin(x)\cos(x) - 2x - y$$

$$\frac{dy}{dx} = \frac{2\sin(x)\cos(x) - 2x - y}{x + 2y}$$

OR $\boxed{\frac{dy}{dx} = \frac{\sin(2x) - 2x - y}{x + 2y}}$

answer: $\boxed{\frac{dy}{dx} = \frac{2\sin(x)\cos(x) - 2x - y}{x + 2y}}$