Score: _____ out of 10.

Math 201 - Quiz #7 Name: key

**Solve as few or as many as you think you need to maximize your score. Please put an X through the problems you do not want graded.**

1. Use the guidelines for curve sketching the produce a sketch (**draw this on the provided window**) for the function

$$f(x) = \frac{x-1}{x}.$$

To receive full credit on this problem you must cover all parts of the guidelines.

1. Domain: $x \neq 0$ i.e., $(-\infty, 0) \cup (0, \infty)$

2. intercepts:

y-intercept: $(0, f(0))$ ← DOES NOT EXIST

x-intercept(s): $\frac{x-1}{x} = 0 \to x-1=0 \to x=1$ so $(1, f(1)) = (1,0)$

3. Symmetry: $f(-x) = \frac{-x-1}{-x} \neq f(x)$ OR $-f(x)$ NO EVEN/ODD Symmetry

4. Asymptotes:

H.A. $\lim_{x\to\infty} \frac{x-1}{x} \overset{L'H}{=} \lim_{x\to\infty} \frac{1}{1} = 1 \to$ H.A. @ $y=1$

$\lim_{x\to-\infty} \frac{x-1}{x} = 1$ ← SAME

V.A. The denominator is 0 if $x=0$ so this is a potential V.A. lets check! ← (remember this is CLOSE too)

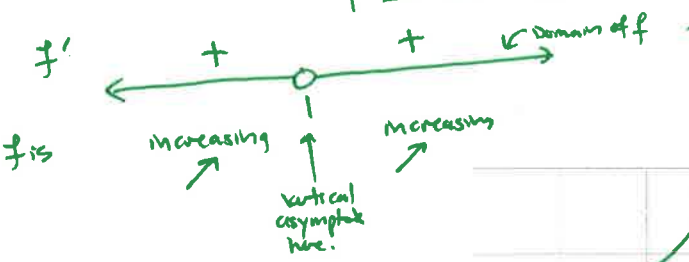
$\lim_{x\to 0^+} \frac{x-1}{x} = -\infty$ ($\frac{-}{+}$)

$\lim_{x\to 0^-} \frac{x-1}{x} = +\infty$ ($\frac{-}{-}$)

5. Sign analysis of $f'$:

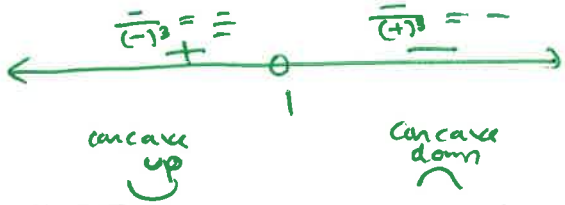
$f'(x) = \frac{x(1)-(x-1)(1)}{(x-1)^2} = \frac{1}{(x-1)^2} = (x-1)^{-2} \to f''(x) = -2(x-1)^{-3} = \frac{-2}{(x-1)^3}$

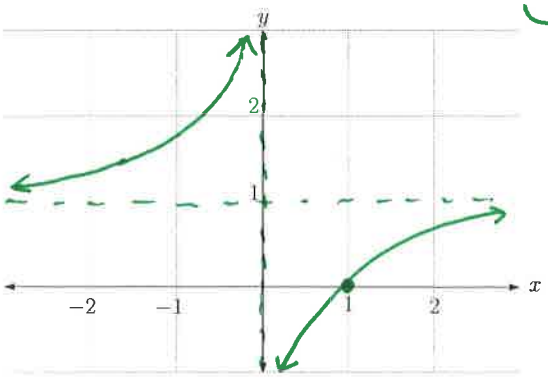
$f'(x)$ is never 0 | $f'(x)$ exists everywhere ON THE DOMAIN




6. Sign analysis of $f''$:

$f''(x)$ is NEVER 0



2. You need to build a running track formed from a rectangle joined with two semicircles as pictured below. The length around the entire track needs to be 2 km. Your design must maximize the area of the rectangular plot enclosed by the track. Find the dimensions of this maximized rectangular plot.


$2 = \text{Track length (perimeter)} = x + \text{half circumference} + x + \text{half circumference}$

$2 = 2x + \text{circumference (with diameter } y)$

$2 = 2x + \pi(\text{diameter})$

$2 = 2x + \pi y \rightarrow y = \frac{2-2x}{\pi}$

we want to ~~maximize~~ the area of the rectangle above:

$$\text{Area} = A = xy$$

Substituting $y = \frac{2-2x}{\pi}$

$$A(x) = x\left(\frac{2-2x}{\pi}\right)$$

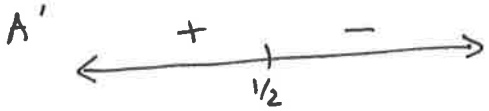
$$A(x) = \frac{2x-2x^2}{\pi} = \frac{2}{\pi}x - \frac{2}{\pi}x^2$$

$$A'(x) = \frac{2}{\pi} - \frac{4}{\pi}x$$

$$A'(x) = 0 = \frac{2}{\pi} - \frac{4}{\pi}x$$

$$\frac{4}{\pi}x = \frac{2}{\pi}$$

$$x = \frac{(2/\pi)}{(4/\pi)} = \frac{2}{\pi}\cdot\frac{\pi}{4} = \frac{1}{2}$$




↑ Absolute max (not just local max) if $x = 1/2$

$$y = \frac{2-2(1/2)}{\pi} = \frac{2-1}{\pi} = \frac{1}{\pi}$$

Dimensions of rectangle: $\boxed{1/2 \text{ km} \times 1/\pi \text{ km}}$