$f(x) = y = 4x^4 - 4x^2$

1. Domain: y is a polynomial $\Rightarrow$ domain is $(-\infty, \infty)$

2. Intercepts:

y-intercept: $(0, f(0)) = (0,0)$

x-intercept: $f(x) = 0$

$4x^4 - 4x^2 = 0$

$4x^2(x^2-1) = 0$

$4x^2(x-1)(x+1) = 0$

$x = 0, \quad x = 1, \quad x = -1$

$\downarrow$

$(0,0), \quad (1,0), \quad (-1,0)$

3. symmetry: $f(-x) = 4(-x)^4 - 4(-x)^2$

$= 4x^4 - 4x^2 = f(x) \Rightarrow f(x)$ is EVEN

4. Asymptotes:

(a) H.A. $\lim_{x\to\infty} 4x^4 - 4x^2 = \infty$

$\lim_{x\to-\infty} 4x^4 - 4x^2 = \infty$

} NO H.A.

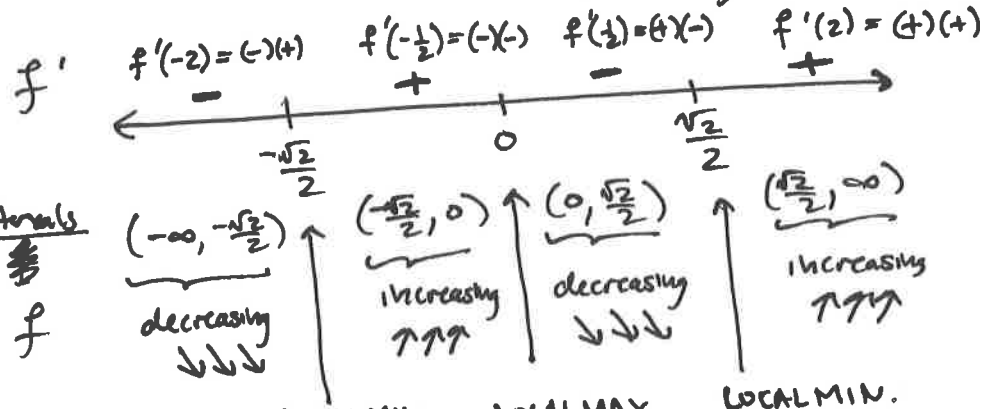
(b) V.A: no V.A.

5. Sign analysis on f':

$f'(x) = 16x^3 - 8x$

$f'(x) = 0 = 8x(2x^2 - 1)$

$x = 0, \quad x = \pm\sqrt{1/2} = \pm\frac{\sqrt{2}}{2}$

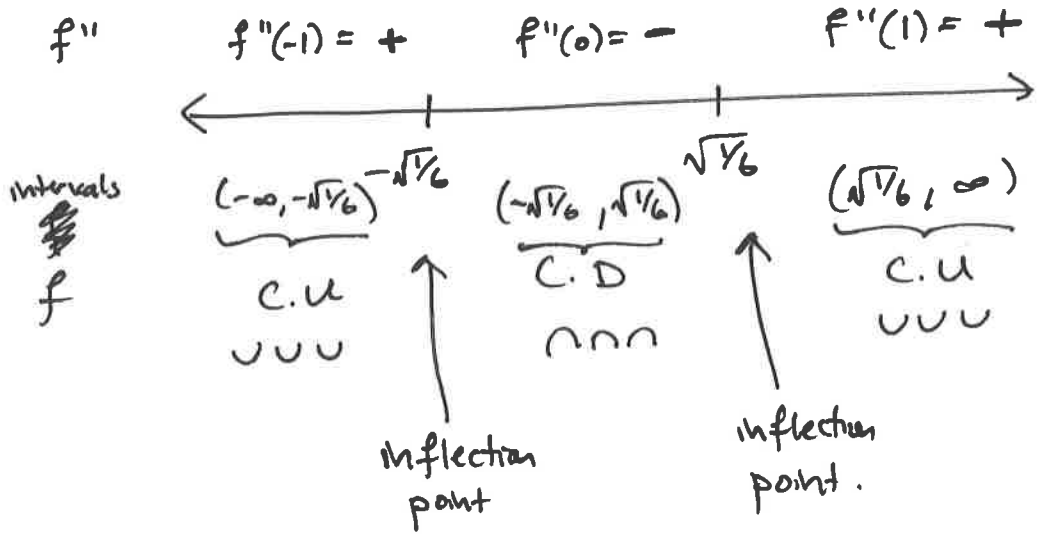


6. Sign analysis on f''

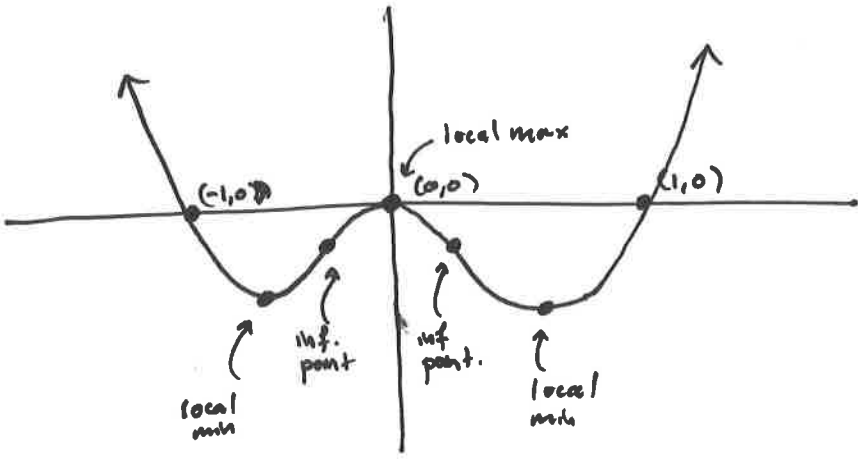
$$f''(x) = 48x^2 - 8 = 8(6x^2 - 1)$$

$$f''(x) = 0 = 6x^2 - 1$$

$$x = \pm\sqrt{\frac{1}{6}}$$



7. Sketch:



Extra: Sometimes you may need to find the coordinates of local max / min and inf. points.