

Key

1. Find  $f(x)$  if  $f''(x) = e^x + \cos(x)$ ,  $f'(0) = 1$ , and  $f(0) = 4$ .

$$\begin{aligned} f'(x) &= e^x + \sin(x) + C \\ f'(0) &= e^0 + \sin(0) + C = 1 \\ 1 + 0 + C &= 1 \\ C &= 0 \\ f'(x) &= e^x + \sin(x) \end{aligned} \quad \rightarrow \quad \begin{aligned} f(x) &= e^x + (-\cos(x)) + D \\ f(0) &= e^0 - \cos(0) + D = 4 \\ 1 - 1 + D &= 4 \\ D &= 4 \\ f(x) &= e^x - \cos(x) + 4 \end{aligned}$$

2. Find the most general antiderivative of the following:

$$\begin{aligned} (a) \quad f(x) &= \frac{5x^2 + x - \sqrt{x} + 1}{x} \\ &= \frac{5x^2}{x} + \frac{x}{x} - \frac{x^{1/2}}{x} + \frac{1}{x} \\ &= 5x + 1 - x^{-1/2} + \frac{1}{x} \end{aligned}$$

$$F(x) = \frac{5x^2}{2} + x - \frac{x^{1/2}}{\frac{1}{2}} + \ln|x| + C = \boxed{\frac{5}{2}x^2 + x - 2\sqrt{x} + \ln|x| + C}$$

$$\begin{aligned} (b) \quad f(x) = 4x^2(x-3)^2 &= 4x^2(x^2 - 6x + 9) \\ &= 4x^4 - 24x^3 + 36x^2 \\ &= 4x^4 - 24x^3 + 36x^2 \end{aligned}$$

$$F(x) = \frac{4x^5}{5} - \frac{24x^4}{4} + \frac{36x^3}{3} + C = \boxed{\frac{4}{5}x^5 - 6x^4 + 12x^3 + C}$$

3. Find  $f(x)$  if  $f'''(x) = 2e^x$ .

$$f''(x) = 2e^x + C$$

$$f'(x) = 2e^x + Cx + D$$

$$f(x) = 2e^x + \frac{Cx^2}{2} + Dx + E$$