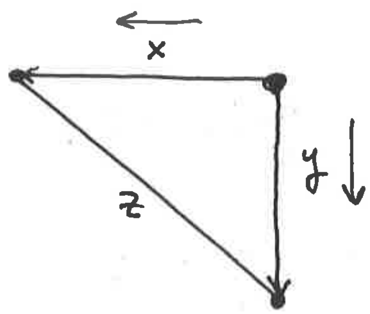
Score: _______ out of 10.

Math 201 - Quiz #5 Name: ______________________________

Solve ONE of the following problems. Please put an X through the problem you do not want graded.

1. Two cars start moving from the same point. One travels south at 60 mi/h and the other travels west at 25 mi/h. At what rate is the distance between the cars increasing two hours later.



given: $\frac{dx}{dt} = 25 \frac{mi}{h}$

$\frac{dy}{dt} = 60 \frac{mi}{h}$

unknown: $\frac{dz}{dt} = ?$ after 2 hours.

equation: $x^2 + y^2 = z^2$

differentiate: $\frac{d}{dt}(x^2 + y^2) = \frac{d}{dt}(z^2)$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{2x\frac{dx}{dt} + 2y\frac{dy}{dt}}{2z} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$

Substitute: After 2 hours $x = 25(2) = 50\text{ mi}.$

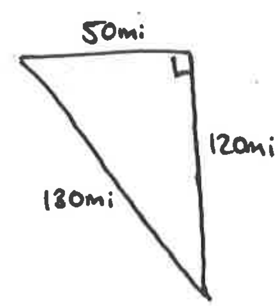
$y = 60(2) = 120\text{ mi}.$

$z = \sqrt{50^2 + 120^2}$

$= \sqrt{25 \cdot 10^2 + 12^2 \cdot 10^2}$

$= \sqrt{(25+144)10^2}$

$= \sqrt{169 \cdot 10^2} = 13 \cdot 10 = 130\text{ mi}$



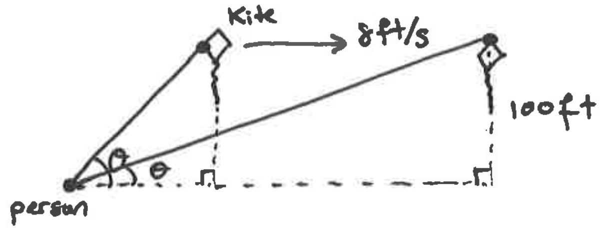
so

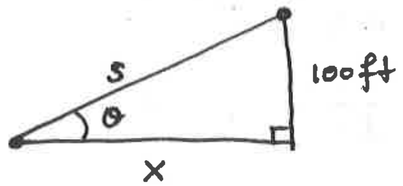
$\frac{dz}{dt} = \frac{50\text{ mi}(25\text{ mi/hr}) + 120\text{ mi}(60\text{ mi/h})}{130\text{ mi}}$

$= \frac{5\text{ mi}(25\text{ mi/hr}) + 12\text{ mi}(60\text{ mi/hr})}{13\text{ mi}}$

$= \frac{125\text{ mi}^2/\text{hr} + 720\text{ mi}^2/\text{hr}}{13\text{ mi}} = \frac{845\text{ mi}^2/\text{hr}}{13\text{ mi}} = \boxed{65 \frac{\text{mi}}{\text{hr}}}$

2. A kite 100 ft above the ground moves horizontally at a speed of 8ft/s. At what rate is the angle between the string and the horizontal decreasing when 200 ft of string has been let out?





given: $\frac{dx}{dt} = 8 \text{ ft/s}$

unknown: $\frac{d\theta}{dt} = ?$ when $s = 200$

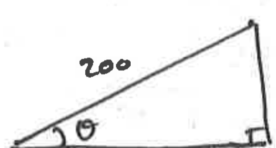
equation: $\cot\theta = \frac{x}{100}$ ← This is the simplest trig equation to use (the other 5 are ok, but this is the best)

$$\frac{d}{dt}\cot\theta = \frac{d}{dt}\left(\frac{x}{100}\right)$$

$$-\csc^2\theta\frac{d\theta}{dt} = \frac{1}{100}\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{\frac{1}{100}\frac{dx}{dt}}{-\csc^2\theta}$$

Substitute:



$\sqrt{200^2 - 100^2}$
$= \sqrt{(2^2-1)100^2}$
$= \sqrt{3}\cdot 100$
$= 100\sqrt{3}$

← (could also notice this is a $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ triangle ($30°, 60°, 90°$ triangle))

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{opp}{hyp}} = \frac{hyp}{opp} = \frac{200}{100} = 2$$

so $$\frac{d\theta}{dt} = \frac{\frac{1}{100 ft}\, 8 \text{ ft/s}}{-(2)^2} = \frac{-2}{100}\text{ rad/s} = \boxed{-\frac{1}{50}\text{ rad/s}}$$