Math 201 - Quiz #6 Name: Key

Pick ONE of the following. Please put an X through the parts you do not want graded.

1. Find the **absolute maximum** and **absolute minimum** values of

$$f(x) = \frac{x}{x^2+1},$$

on the interval $[0, 2]$.

SOL: Here we can use the closed interval method.

$$f'(x) = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

Critical numbers:

$$f'(x) = 0 \qquad 1-x^2 = 0 \qquad 1 = x^2 \qquad x = \pm 1$$

only $x=1$ is in $[0,2]$

$f'(x) =$ DNE if $(x^2+1)^2 = 0$

NO REAL SOLUTIONS.

$f(1) = 1/2$

$f(0) = 0 \leftarrow$ Absolute MIN value (in $[0,2]$)

$f(2) = 2/5 \leftarrow$ Absolute MAX value (in $[0,2]$)

2. Find the **absolute maximum** and **absolute minimum** values of

$$f(t) = 2\cos(t) + 2\sin(t),$$

on the interval $[0, 2\pi]$.

SOL: Closed interval method:

$$f'(t) = -2\sin(t) + 2\cos(t)$$

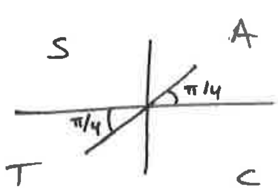
critical numbers: (since $f'(t)$ always exists just SOLVE $f'(t)=0$)

$$f'(t) = 0 = -2\sin(t) + 2\cos(t)$$

$$\cancel{2}\sin(t) = \cancel{2}\cos(t)$$

$$\frac{\sin(t)}{\cos(t)} = 1$$

$$\tan(t) = 1$$



$$t = \frac{\pi}{4}, \quad \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

$f\left(\frac{\pi}{4}\right) = \frac{\cancel{2}\sqrt{2}}{\cancel{2}} + \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = 2\sqrt{2} \leftarrow$ ABS MAX $\quad f(0) = 2\cos(0) + 2\sin(0) = 2$

$f\left(\frac{5\pi}{4}\right) = -\frac{\cancel{2}\sqrt{2}}{\cancel{2}} - \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = -2\sqrt{2} \leftarrow$ ABS MIN $\quad f(2\pi) = 2$

3. Find all numbers c that satisfy the conclusion of the Mean Value Theorem for $f(x) = e^{-2x}$ on $[0, 3]$.

$f'(x) = e^{-2x}(-2)$

MVT on $[0,3]$: there exists a c in $(0,3)$ s.t.

$$f'(c) = \frac{f(3) - f(0)}{3 - 0}$$

$$-2e^{-2c} = \frac{e^{-6} - e^{0}}{3} = \frac{e^{-6} - 1}{3}$$

$$e^{-2c} = \frac{e^{-6} - 1}{-6}$$

$$\ln\left(e^{-2c}\right) = \ln\left(\frac{e^{-6} - 1}{6}\right)$$

$$-2c = \ln\left(\frac{e^{-6} - 1}{6}\right) \longrightarrow \boxed{c = -\frac{1}{2}\ln\left(\frac{e^{-6} - 1}{6}\right)}$$

4. Suppose $3 \le f'(x) \le 5$ for all values of x. Show that $18 \le f(8) - f(2) \le 30$.

This is a nice challenge problem.

By MVT: there exists a c in $(2,8)$ s.t.

$$f'(c) = \frac{f(8) - f(2)}{8 - 2} = \frac{f(8) - f(2)}{6}$$

that is

$$6f'(c) = f(8) - f(2)$$

since $3 \le f'(x) \le 5$ we know

$$6 \cdot 3 \le 6f'(c) \le 6 \cdot 5$$

$$18 \le \underbrace{6f'(c)} \le 30$$

Hence, $18 \le f(8) - f(2) \le 30$